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Fractional Modeling of the Gravitational Potential in Galactic Bulges with Exp Profile

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Abstract. We explore a modified-gravity framework for modeling the bulge contribution to spiral galaxy rotation curves by generalizing the Poisson equation through a Riesz fractional derivative. An analytic fractional gravitational potential is derived for an exponential density profile and applied to the Milky Way rotation curve, modeled as the superposition of an exponential disk, a fractional bulge, and an NFW dark matter halo, using Bayesian inference. The results favor a fractional index $s \simeq 0.44$, consistent with a smoother central potential and a pseudo-bulge, while the halo parameters remain compatible with standard NFW expectations, indicating that fractional operators provide a viable extension to galactic bulge gravitational modeling.

Resumo. Exploramos um modelo de gravidade modificada para o bojo galáctico, generalizando a equação de Poisson por meio de uma derivada fracionária de Riesz. Derivamos um potencial fracionário analítico para um perfil exponencial e o aplicamos à curva de rotação da Via Láctea, modelada com disco exponencial, bojo fracionário e halo NFW, utilizando inferência Bayesiana. Os resultados favorecem um índice fracionário $s \simeq 0{,}44$, compatível com um potencial central mais suave e com um pseudo-bojo, mantendo parâmetros do halo consistentes com o cenário NFW padrão, indicando que operadores fracionários constituem uma extensão viável da modelagem gravitacional de bojos galácticos.

Keywords. Galactic bulges

1. Introduction

Spiral galaxies can be decomposed into 3 dynamical components: Halo, Disk and Bulge. Different galaxies have distinct morphologies and bulge profiles, although it is common to define them by a Hernquist or exponential profile, these models may not capture the dynamical profile of many galactic bulges. In this regard, it is quite common to use Sérsic's Law to adjust the circular component of the bulge.

The objective of this work is to generalize the gravitational Poisson equation by replacing the classical Laplacian with the Riesz fractional derivative, defined in terms of the Fourier transform. This approach falls within the scope of the theory of fractional operators, which have been gaining ground as natural extensions of Physics to describe non-local phenomena and long-range effects, potentially relevant on the galactic scale.

We obtained a fractional formulation for the potential applied to the Exp profile whose order of the derivative allows us to modify the behavior of the curve.

2. The Fractional Potential

We consider the fractional gravitational potential $\phi(r)$ corresponding to an exponential density profile

$$\tau^{(2s-2)R} D_t^s \phi(r) = 4\pi G \rho_0 e^{-ar}, \quad (1)$$

where $\tau^{(2s-2)}$ dimensional correction factor (Kuroda, 2021), ${}^R D_t^s$ Riesz fractional derivative (Herrmann, 2011) and boundary condition $\lim_{r\to\infty} \phi(r) = 0$. The solution is:

$$\Phi(r) = -\tau^{2-2s} \frac{16aG\rho_0}{r} \int_0^\infty \frac{\sin(kr)}{k^{2s-1}(a^2+k^2)^2}\, dk, \qquad s \in (0,1] \quad (2)$$

This result gives the analytic form of the gravitational potential generated by an exponential density profile in three dimensions.

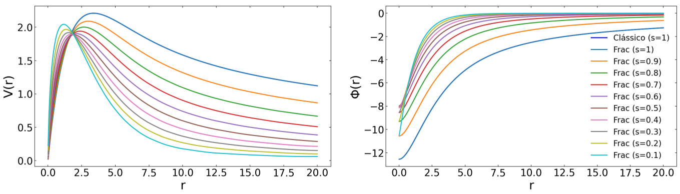

Figure 1. Left: Rotation curve of the fractional gravitational potential. when the index of the derivative is 1 we have the circular velocity of the classical case of the exponential sphere. Rigth: Fractional gravitational potential. when the index of the derivative is 1 we have the classic case of the exponential sphere, the lines represent different curves for different values of s.

We model the Milky Way rotation curve as the superposition of three mass components: an exponential disk, a fractional bulge, and a dark matter halo.

We fix the stellar disk to an exponential surface density profile (Freeman, 1970),with scale length $R_d = 5.4$ kpc and disk mass $M_d = 6 \times 10^{10} M_\odot$ (values supported by Huang et al, 2016 and Karukes et al, 2020). The corresponding circular velocity is computed using Bessel functions I_n, K_n.

We adopt a Navarro–Frenk–White (NFW) density profile (Navarro, Frenk & White, 1997), from which the enclosed mass is computed to determine the halo contribution to the rotation curve.

The bulge potential is modeled with a Riesz Fractional derivative, motivated by generalized Poisson equations. We evaluate the velocity contribution via a precomputed lookup table for the function interpolated with cubic splines or bilinear schemes; if unavailable, we fall back to direct numerical integration. Free bulge parameters are: bulge mass M_b, scale a, and fractional index s.

Table 1. Parameters of the models

Parameter	Range (Prior)	Description
$\log_{10}(M_b/M_\odot)$	[8.0, 10.5]	Bulge mass
a [kpc]	[0.1, 6.0]	Bulge scale
s	[0.1, 1.0]	Fractional index
$\log_{10}(\rho_s)$ [$M_\odot$/pc^3]	[−6.0, 2.0]	NFW scale density
$\log_{10}(r_s$/kpc)	[−1.0, 1.8]	NFW scale radius

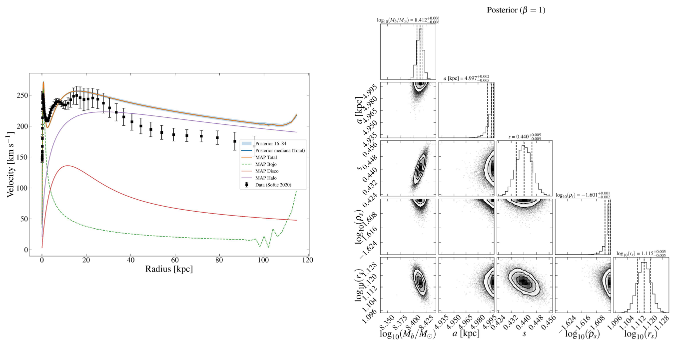

Figure 2. Left: Rotation curve of the Milky Way fitted by the model described here. The red line represents the exponential disk curve, the purple curve the NFW halo, the dashed green curve the fractional potential profile. For R>90, we have numerical instability due to a low contribution from the bulge at large radii. Rigth: Corner plots of the correlation between the posterior distributions of the fitted parameters. It is possible to notice that some parameters have a cut, this occurs because the best value found, that is, the minimum is at the limit of the adopted prior.

We fit the observed rotation curve data of Sofue (2020) using Bayesian statistics. The likelihood is Gaussian with observational errors, and the prior distributions are uniform over broad physical ranges (see table). To explore the posterior, we employ the affine-invariant ensemble sampler `emcee` with multiple move strategies (stretch, differential evolution, snooker, KDE).

To compute the Bayesian evidence and handle multimodal posteriors, we apply thermodynamic integration (TI) across a ladder of inverse temperatures β. The parallelized MPI implementation distributes β blocks across processes, ensuring convergence and robustness.

This combined framework allows us to test fractional gravity models in the bulge while constraining the disk and halo components consistently with observational data. The use of TI provides robust evidence

3. Results

The Milky Way rotation curve (Sofue 2020) combines several tracers in different radial regions, recalibrated to the same galactic values, generating a continuous profile from 0.1 to 100 kpc. The core is anchored by the stellar orbits in SgrA*, with CO and HI tracing the gas. In the inner disk, the terminal velocity of HI and CO defines the classical curve, refined by HII regions, OB stars, VLBI masers, Gaia data, and spectroscopic surveys. In the outer regions, halo stars, globular clusters, satellites, and the disk thickness HI are used. All data are homogeneous in (R_0, V_0) and combined by a weighted moving average, resulting in the final continuous curve.

We summarize below the global maximum a posteriori (MAP) estimates, highest posterior density (HPD) intervals, and global metrics for the Milky Way rotation curve fit.

Table 2. Results from the fitting

Parameter	MAP	HPD 68%	HPD 95%
$\log_{10}(M_b/M_\odot)$	8.41	[8.41, 8.42]	[8.40, 8.42]
a [kpc]	5.00	[4.99, 5.00]	[4.99, 5.00]
s	0.44	[0.44, 0.45]	[0.43, 0.45]
$\log_{10}(\rho_s)$	-1.60	[-1.60, -1.60]	[-1.61, -1.60]
$\log_{10}(r_s)$	1.11	[1.11, 1.12]	[1.11, 1.12]

Global metrics: $\log Z = -694.4$ (thermodynamic integration), BIC = 1358.8, WAIC = 1782.9, and χ^2/dof ≈ 19.7.

The fit reveals a bulge mass of $\sim 2.6 \times 10^8 M_\odot$ with a scale length tightly constrained near $a \simeq 5$ kpc and a fractional index $s \simeq 0.44$, consistent across multiple posterior modes. The NFW halo parameters converge to $\rho_s \sim 0.025$ $M_\odot$/pc^3 and $r_s \sim 13$ kpc. Despite the presence of three posterior modes with comparable weights (~0.25–0.49), all modes agree within uncertainties, reflecting a robust inference. The relatively high χ^2/dof indicates residual tensions between the model and the compiled data of Sofue (2020), likely arising from systematics in heterogeneous tracers rather than parameter degeneracy.

4. Conclusions

The fit to the full Milky Way rotation curve is unsatisfactory, as indicated by a χ^2/dof significantly larger than unity, suggesting systematic discrepancies between the model and the heterogeneous data. These discrepancies may arise from residual observational systematics, simplifying assumptions in the disk and halo models, or the limitations of parametric descriptions of the Galaxy.

From a theoretical standpoint, this work is framed within modified gravity, where the gravitational potential is governed by a fractional Laplacian rather than the standard Poisson equation. This non-local modification introduces a scale-dependent dynamics that can effectively mimic additional mass components.

Finally, the Milky Way bulge is known to be a composite structure, comprising a box/peanut bar, a nuclear stellar disk, and possibly a small classical bulge component, rather than a single monolithic system.

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