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Fitting the Sersic Profile to the Milky Way Bulge via Bayesian Sampling

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Abstract. We analyze the Milky Way rotation curve using Bayesian inference to test how different bulge parameterizations affect the inferred central mass distribution and dark matter halo properties. Three bulge models are considered, Sérsic 2D, Sérsic 3D (Prugniel–Simien), and Einasto, combined with either a cuspy NFW or a cored isothermal halo, while disk parameters are fixed. Using MCMC fits to the Sofue (2020) rotation-curve data, we find that all bulge models converge to Sérsic/Einasto indices close to unity, indicating a robust pseudo-bulge structure largely independent of the adopted density law. Bayesian model comparison decisively favors the NFW halo over the isothermal one, with large differences in BIC and Bayes factors, as the cored profile fails to reproduce the outer rotation curve. Our results show that the Milky Way bulge is nearly exponential and that the global kinematics strongly support a cuspy dark matter halo, in agreement with cosmological expectations.

Resumo. Analisamos a curva de rotação da Via Láctea usando inferência Bayesiana para testar como diferentes parametrizações do bojo afetam a distribuição de massa central inferida e as propriedades do halo de matéria escura. Três modelos de bojo são considerados, Sérsic 2D, Sérsic 3D (Prugniel–Simien) e Einasto, combinados com um halo NFW com cúspide ou um halo isotérmico com núcleo, enquanto os parâmetros do disco são mantidos fixos. Usando ajustes MCMC aos dados da curva de rotação de Sofue (2020), descobrimos que todos os modelos de bojo convergem para índices Sérsic/Einasto próximos da unidade, indicando uma estrutura de pseudo-bojo robusta, em grande parte independente da lei de densidade adotada. A comparação Bayesiana dos modelos favorece decisivamente o halo NFW em relação ao isotérmico, com grandes diferenças nos fatores BIC e Bayes, visto que o perfil com núcleo não consegue reproduzir a curva de rotação externa. Nossos resultados mostram que o bojo da Via Láctea é quase exponencial e que a cinemática global apoia fortemente um halo de matéria escura com cúspide, em concordância com as expectativas cosmológicas.

Keywords. Galactic bulge – Milky Way

1. Introduction

The Sérsic profile is a versatile model for the brightness distribution of elliptical galaxies, classical bulges, and pseudo-bulges (nuclear discs). Its index n governs the shape: $n = 1$ yields an exponential law and $n = 4$ recovers the de Vaucouleurs profile. Beyond the 2D formulation, 3D generalizations are routinely used: the Prugniel–Simien (PS) approximation provides a 3D density consistent with projected Sérsic, and the *Einasto* law, mathematically akin to Sérsic, is common in simulations of baryons and dark matter.

The Milky Way bulge is composite, combining the box/peanut (bar) structure, a flattened nuclear stellar disc, and possibly a small nearly-spherical old component. The first two are expected to have Sérsic indices near unity, consistent with pseudo-bulges.

Here we assess which bulge parameterization (Sérsic 2D, Sérsic 3D/PS, or Einasto) best describes the Milky Way central mass distribution when coupled with different dark matter halos (NFW and cored isothermal). We perform Bayesian inference with MCMC (48 walkers, 50k steps) using the rotation-curve data of Sofue (2020). Disk parameters are fixed ($R_d = 5.4$ kpc, $M_d = 6 \times 10^{10} M_\odot$); bulge and halo parameters are free.

2. Methodology

Bulge: Three alternative density/brightness laws are tested:

- *Sérsic 2D (surface brightness):*

$$I(R) = I_e \exp\left\{-b_n\left[\left(\frac{R}{R_e}\right)^{1/n} - 1\right]\right\}. \tag{1}$$

Table 1. Parameters used in the models

Component	Parameter	Status	Prior
Bulges	$\log_{10}(M_b/M_\odot)$	Free	[8.0, 10.0]
	R_e or r_{-2} [kpc]	Free	[0.01, 10.0]
	n	Free	[0.5, 5.0]
NFW	$\log_{10}(\rho_s/\mathrm{M_\odot\,pc^{-3}})$	Free	[-3.0, 1.0]
	$\log_{10}(r_s/\mathrm{kpc})$	Free	[0.0, 2.0]
ISO	$V_{\max}$ [km/s]	Free	[100, 300]
	r_c [kpc]	Free	[0.1, 20.0]
Disk	M_d $[M_\odot]$	Fixed	6×10^{10}
	R_d [kpc]	Fixed	5.4

- *Sérsic 3D (Prugniel–Simien):*

$$\rho(r) = \rho_e \left(\frac{r}{R_e}\right)^{-p_n} \exp\left\{-b_n\left[\left(\frac{r}{R_e}\right)^{1/n} - 1\right]\right\}, \tag{2}$$

with $p_n \simeq 1 - 0.6097/n + 0.05563/n^2$ ensuring consistency with the projected Sérsic law.

- *Einasto:*

$$\rho(r) = \rho_{-2} \exp\left\{-\frac{2}{n}\left[\left(\frac{r}{r_{-2}}\right)^{n} - 1\right]\right\}, \tag{3}$$

where r_{-2} is the radius at which the logarithmic slope equals -2.

These parameterizations test whether the inferred bulge structure depends on the assumed density law.

Halo: Two models:

Table 2. Final results

	Sérsic 2D		Sérsic 3D (PS)		Einasto	
	NFW	ISO	NFW	ISO	NFW	ISO
$\log_{10}(M_b/M_\odot)$	9.96	9.92	9.95	9.90	9.96	9.92
R_e or r_{-2} [kpc]	0.214	0.202	0.265	0.235	0.356	0.314
n (index)	1.10	0.96	0.87	0.79	1.10	0.95
$\log_{10}(\rho_s)$	-1.14	–	-1.12	–	-1.14	–
$\log_{10}(r_s)$ [kpc]	0.84	–	0.83	–	0.84	–
$V_{\max}$ [km/s]	–	196	–	196	–	196
r_c [kpc]	–	0.69	–	0.65	–	0.69
$\log \mathcal{L}_{\max}$	-23.9	-42.4	-25.2	-42.0	-23.9	-42.4
BIC	69.2	106.2	71.9	105.5	69.2	106.2
$\log Z$ (Lapl.)	-49.2	-67.3	-51.1	-64.5	-48.9	-67.0

– *NFW:*

$$\rho_{\rm NFW}(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}, \quad (4)$$

The NFW (Navarro, Frenk & White, 1997) profile describes a "cuspy" dark matter halo. Its density, $\rho(r)$, diverges as $r \to 0$ and falls off as r^{-3} at large radii.

– *Cored isothermal (ISO):*

$$\rho_{\rm ISO}(r) = \frac{\rho_0}{1+\left(\frac{r}{r_c}\right)^2}. \quad (5)$$

The cored isothermal sphere (ISO) profile models a dark matter halo with a constant central density, i.e., a "core" rather than a cusp. Its density falls off as r^{-2} at large radii, $r \gg r_c$.

We used different halo profiles, a core and a cusp, not only to determine the best DM model for the Milky Way, but to verify the impact of the central density of the halo on the rotational profile of the bulge.

Disk: Exponential disk fixed ($M_d = 6 \times 10^{10} M_\odot$, $R_d = 5.4$ kpc; Karukes et al. 2020). The circular velocity uses modified Bessel functions I_0, I_1, K_0, K_1 (Freeman 1970).

Inference: We first run *Differential Evolution* to avoid local minima, then sample with `emcee` (48 walkers, 10k burn-in, 50k steps). Gaussian, independent errors:

$$\log \mathcal{L}(\theta) = -\frac{1}{2}\sum_i \frac{[v_i - v_{\rm tot}(r_i|\theta)]^2}{\sigma_i^2}. \quad (6)$$

Uniform priors within physically motivated bounds (see Huang et al. 2016; Karukes et al. 2020).

We report posterior modes (KDE) and 68% HDIs. Model comparison uses BIC,

$$\mathrm{BIC} = k \ln n - 2 \log \mathcal{L}_{\max}, \quad (7)$$

and Laplace-approximated evidence,

$$\log Z \approx \log \mathcal{L}(\hat{\theta}) + \log \pi(\hat{\theta}) + \frac{k}{2}\log(2\pi) + \frac{1}{2}\log|\Sigma|. \quad (8)$$

Which is not a problem since the posteriors have Gaussian behavior. The Bayes factor $K = e^{\Delta \log Z}$ quantifies preference between NFW and ISO halos.

3. Results

The bulge parameters are robust across all three parameterizations ($n \approx 1$), consistent with a pseudo-bulge structure. However, model comparison decisively favors the NFW halo over the isothermal one: ΔBIC $\sim$ 34–37 and Bayes factors $K \sim 10^{-6}$–10^{-8}, reflecting the failure of the ISO profile to reproduce the outer rotation curve.

4. Conclusions

We analyzed the Milky Way rotation curve using Bayesian inference with MCMC, testing three bulge parameterizations (Sérsic 2D, Sérsic 3D/PS, and Einasto) combined with two halo models (cuspy NFW and cored isothermal).

The Sérsic/Einasto indices converge to values close to unity, which indicates a pseudo-bulge–like structure. The bulge mass is recovered around $M_b \sim (8\text{–}9) \times 10^9\, M_\odot$, and the effective radius remains compact ($R_e \sim 0.2$–0.3 kpc). This robustness shows that the characterization of the Milky Way bulge is essentially independent of the functional form of the density law.

While the NFW halo reproduces the rotation curve well from the inner to the outer regions, the cored isothermal profile systematically fails at $r \gtrsim 30$–35 kpc, where it overestimates the observed velocities. Quantitatively, Bayesian model selection yields ΔBIC $\sim$ 34–37 and Bayes factors $K \sim 10^{-6}$–10^{-8} against the ISO model, which constitutes overwhelming evidence in favor of the NFW halo. The Bayesian evidence is consistent across all bulge profiles, reinforcing that this conclusion is not sensitive to the bulge parameterization.

Therefore, we conclude that:

– the Milky Way bulge is robustly constrained to be nearly exponential ($n \approx 1$), independent of the adopted functional form. This is consistent with the combined presence of a box/peanut (bar) and a nuclear disc, and the absence of a significant classical bulge;

– the global rotation curve analysis decisively supports a cuspy halo over a cored one;

– the results are fully consistent with cosmological expectations for galaxies of similar stellar and halo mass.

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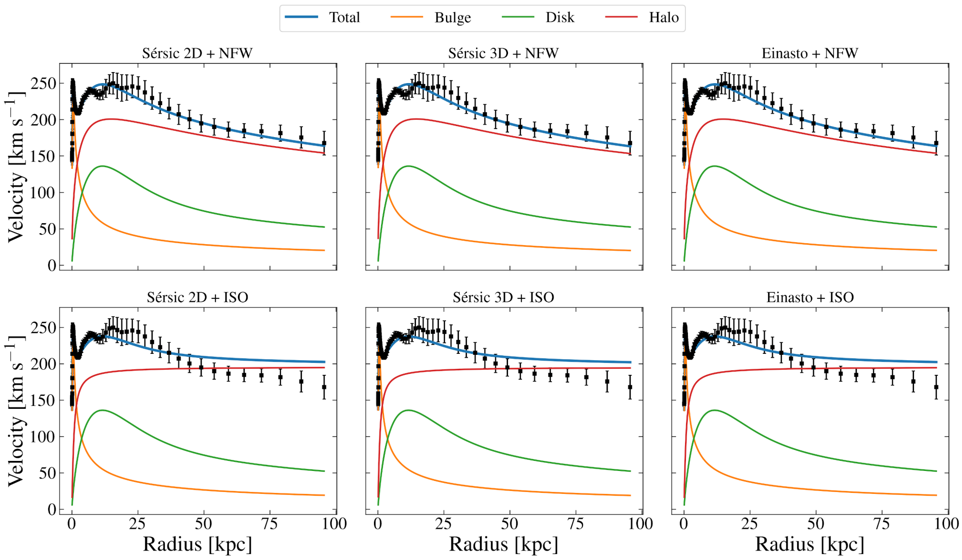

Figure 1. Rotation curves for the six combinations: Sérsic 2D, Sérsic 3D (PS), and Einasto, each with NFW and ISO. Data from Sofue (2020).

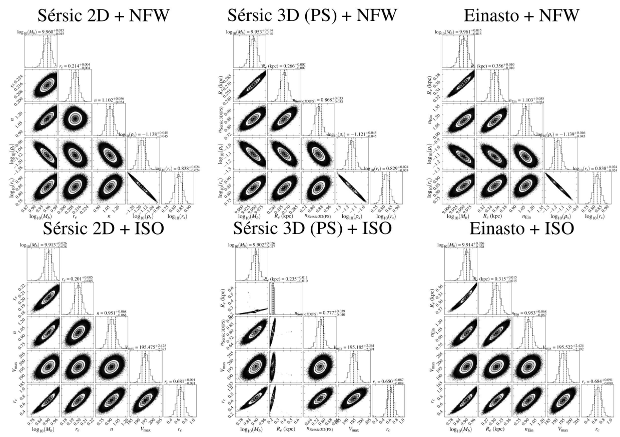

Figure 2. Posterior distributions (corner plots) for all six models. NFW solutions are tight and consistent; ISO posteriors are broader and less compatible.

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