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Gravitational redshift, Shapiro delay, and the shadow of a Schwarzschild black hole with corrections given by dark photons

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Abstract. We investigate gravitational redshift and Shapiro delay in a black hole metric with dark photon corrections (Yukawa and magnetic dipole) proposed by Övgün and Pantig (2025). We derive analytical corrections, showing they are exponentially suppressed by the dark photon mass, constraining the dark sector only in high-precision weak-field tests.

Resumo. Investigamos o redshift gravitacional e o atraso de Shapiro em uma métrica de buraco negro com correções de fóton escuro (Yukawa e dipolo magnético) de Övgün e Pantig (2025). Derivamos correções analíticas, demonstrando supressão exponencial pela massa do fóton escuro, restringindo o setor escuro apenas em testes de campo fraco de alta precisão.

Keywords. Black hole physics – Gravitation – Gravitational lensing: strong

1. Introduction

General Relativity (GR) is robust, but the presence of dark matter suggests a "dark sector" that may interact via new mediators. Recent studies have explored how dark matter halos and quantum corrections modify spacetime geometry and black hole shadows (Konoplya 2019; Jusufi et al. 2019). Furthermore, phenomenological investigations in stellar metrics suggest subtle signatures of quantum gravity in classical observables (Pantig et al. 2025).

In this context, Övgün & Pantig (2025) obtained a black hole solution incorporating dark photon corrections (minimal and dipole interactions). This work extends this analysis to the weak field regime, calculating signatures in gravitational redshift and Shapiro delay.

2. Metric and Approximations

We consider a static and spherically symmetric spacetime $ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$. In the model of Övgün & Pantig (2025), for large distances ($m_{A'} r \gg 1$), the metric function is:

$$f(r) \approx 1 - \frac{2M}{r} + \alpha \frac{e^{-mr}(mr+1)}{r} - C\frac{e^{-mr}}{r^2}, \quad (1)$$

where we define the dark photon mass as $m \equiv m_{A'}$. The coupling constants for the minimal interaction (α) and dipole (C) are defined, respectively, as:

$$\alpha \equiv \frac{g_D^2}{2\pi}, \qquad C \equiv \frac{\mu_f^2 S_{12} m}{2\pi\Lambda^2}. \quad (2)$$

These parameters control the intensity of corrections relative to Newtonian/Einsteinian gravity.

Figura 1. Comparison between Schwarzschild and Dark Photon model for different $m_{A'}$.

3. Weak Field Tests

3.1. Gravitational Redshift

The redshift z for an emitter at r_e is $z = f(r_e)^{-1/2} - 1$. In the weak field regime, we expand in Taylor series $(1-x)^{-1/2} \approx 1 + x/2$. The analytical expression for the deviation becomes:

$$z \approx \frac{M}{r_e} \underbrace{- \frac{\alpha}{2r_e} e^{-mr_e}(mr_e + 1) + \frac{C}{2r_e^2} e^{-mr_e}}_{\delta z_{\rm dark}}. \quad (3)$$

The first term is the classical GR one. The corrections ($\delta z_{\rm dark}$) depend on e^{-mr_e}, indicating that for non-negligible m, the effects are local. Fig. 2 illustrates the fractional deviation.

3.2. Shapiro Time Delay

The time delay Δt for a reflected signal (round trip $r_1 \to r_0 \to r_1$) is given by the integral $\Delta t = \int (f(r)^{-1} - 1)dz$, with the trajectory approximated by $r(z) \approx \sqrt{z^2 + r_0^2}$.

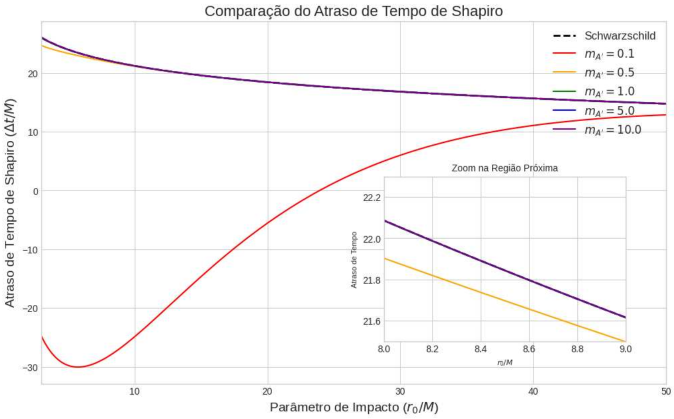

Figura 2. Relative difference $\Delta z/z_{GR}$ as a function of r_e.

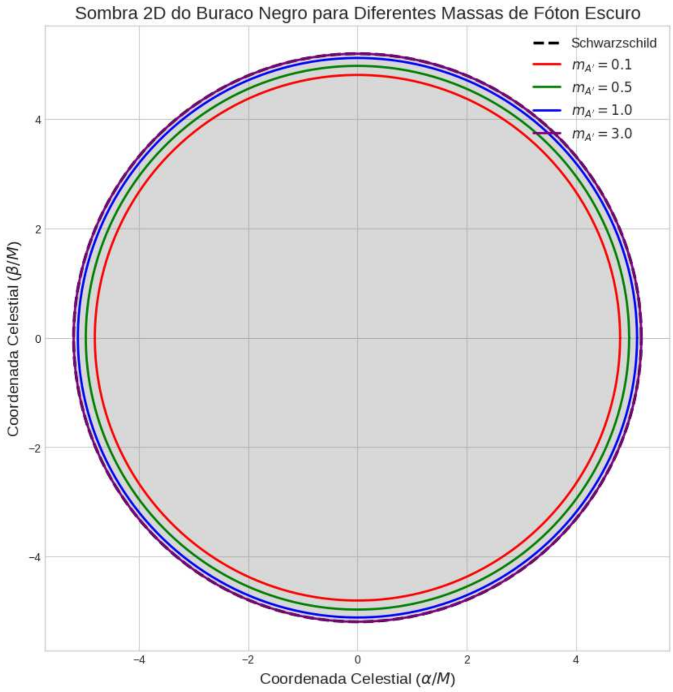

Figura 3. Time correction Δt_{corr} vs impact parameter r_0.

Approximating $f(r)^{-1} \approx 1 - (f(r) - 1)$ and integrating, we obtain:

$$\Delta t \approx 4M \ln\left(\frac{4r_1}{r_0}\right) - 2r_1 e^{-mr_0}\left[\frac{\alpha}{r_0}(mr_0 + 1) - \frac{C}{r_0^2}\right]. \quad (4)$$

We assume $r_1 \approx r_2 \gg r_0$. The logarithmic term is the standard GR delay. The linear term in r_1 in the corrections arises from the integration of the Yukawa potential, but is strongly suppressed by the exponential factor e^{-mr_0} if r_0 is large compared to the dark photon Compton wavelength $(1/m)$.

4. Conclusion

We calculated the first-order corrections for redshift and Shapiro delay. The presence of the factor e^{-mr} in both observables implies that massive dark photons ($m \gtrsim$ eV) do not produce detectable signals on Solar System scales. However, for ultralight dark photons ($m \to 0$), the algebraic correction terms become relevant, allowing precision experiments to impose upper limits on the couplings α and C.

References

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