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Is $\omega_0\omega_a$CDM a good model for the clumpy universe?

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Abstract. The DESI collaboration recently obtained a set of precise BAO measurements, that combined with CMB and SNIa datasets, show that the $\omega_0\omega_a$CDM model is preferred over ΛCDM, at more than $4\,\sigma$, to describe the dynamics of the expanding universe. This raises the question whether this model also suitably describes the clumpy universe. Also lately, detailed analyses of diverse cosmic tracers resulted in a new dataset of measurements of an observable from the clumpy universe: $\sigma_8(z)$, spanning a high-redshift data $z \in [0.013, 3.8]$. In this work we use this dataset of 15 $\sigma_8(z_i)$ measurements to study the viability of the $\omega_0\omega_a$CDM cosmological model to explain the clustered universe. Our analyses compare the $\omega_0\omega_a$CDM model with the $\sigma_8(z)$ function reconstructed from the data points using Gaussian Process. Moreover, we perform a similar evaluation of the ΛCDM model considering Planck and DESI best-fit parameters.

Resumo. A colaboração DESI recentemente obteve um conjunto de dados precisos de OAB que, combinados com dados de RCF e SNIa, mostram que o modelo $\omega_0\omega_a$CDM é preferido em relação ao ΛCDM, em mais de $4\,\sigma$, para descrever a dinâmica do universo em expansão. Este resultado levanta o questionamento se este modelo também descreve apropriadamente o universo perturbado. Coincidentemente, análises detalhadas de diversos traçadores cósmicos resultaram em um novo conjunto de dados de um observável do universo perturbado: $\sigma_8(z)$, cobrindo um intervalo de redshift de $z \in [0.013, 3.8]$. Neste trabalho nós usamos esse conjunto de 15 medidas de $\sigma_8(z_i)$ para estudar a viabilidade do modelo $\omega_0\omega_a$CDM para explicar o universo perturbado. Nossas análises comparam o modelo $\omega_0\omega_a$CDM com a sua função reconstruída de $\sigma_8(z)$ a partir dos dados utilizando Processos Gaussianos. Além disso, nós realizamos uma análise similar do modelo ΛCDM considerando os valores dos parâmetros obtidos pelas colaborações Planck e DESI.

Keywords. large-scale structure of Universe – dark energy – cosmological parameters

1. Introduction

Precise measurements of the Baryon Acoustic Oscillations (BAO) have been recently obtained by the Dark Energy Spectroscopic Instrument (DESI) collaboration (DESI 2025), suggesting a turning point in the determination of the standard cosmological model. Considering the Chevallier-Polarski-Linder (CPL) parametrization (Chevallier 2000; Linder 2002) for the time-dependent dark energy equation of state, $\omega(a) = \omega_0 + \omega_a\,(1 - a)$, where a is the space-time scale factor normalized to $a = 1$ at present time, the DESI analyses combined BAO, Cosmic Microwave Background (CMB), and Type Ia Supernovae (SNIa) data to find a highly significant preference for the $\omega_0\omega_a$CDM model as compared with the ΛCDM.

This result shows that this model suitably describes the accelerated expansion of the background universe and raises the question if the $\omega_0\omega_a$CDM model also explains satisfactorily the data from the clumpy universe. For this purpose, one need to study its viability with observables from the clustered universe, such as f and $f\sigma_8$ (Avila 2022). Although these data are not as precise as one would like, they are still suitable for testing the viability of alternative models (Oliveira 2025a; Basilakos 2017; Ribeiro 2024).

In recent years, there have been efforts to measure, at several redshifts, another important cosmic observable from the clustered universe: $\sigma_{8,0} \equiv \sigma_8(z = 0)$ the present-day matter fluctuations amplitude at the scale of 8 Mpc$/h$. As a result, currently we have a set of 15 measurements in the redshift interval $z \in [0.013,\ 3.80]$ (Piccirilli 2024; Franco 2025a).

Our methodology to investigate if the $\omega_0\omega_a$CDM model describes suitably the data from the clumped universe is developed in two steps: (i) we use Gaussian Processes (GP) to reconstruct in a model-independent way the function $\sigma_8(z)$, termed $\sigma_8^{\rm rec}(z)$, from this set of 15 $\sigma_8(z_i)$ measurements; (ii) we examine if the $\omega_0\omega_a$CDM model obtained from DESI analyses properly fits the function $\sigma_8^{\rm rec}(z)$. This statistical evaluation includes the comparison of the function $\sigma_8^{\rm rec}(z)$ with the ΛCDM model obtained considering the best-fit parameters found by the Planck and DESI analyses.

2. Methodology

The evolution of the cosmological observable $\sigma_8(z)$ is described using the linear cosmological perturbation theory, which governs the growth density fluctuations through the matter density contrast $\delta_m(\mathbf{r}, a)$, with $a(t)$ the scale factor, or equivalently $\delta_m(\mathbf{r}, t)$, dependent on the cosmic time t, and defined as

$$\delta_m(\mathbf{r}, t) \equiv \frac{\rho_m(\mathbf{r}, t) - \bar{\rho}_m(t)}{\bar{\rho}_m(t)}\,, \tag{1}$$

where $\rho_m(\mathbf{r}, t)$ is the matter density at position $\mathbf{r}$ and cosmic time t, and $\bar{\rho}_m(t)$ is the background matter density at the same epoch.

In the Newtonian approach, i.e., for sub-horizon scales, one can obtain a second order differential equation to describe the matter fluctuations (Coles 1996)

$$\ddot{\delta}_m(t) + 2H(t)\,\dot{\delta}_m(t) - 4\pi G\,\bar{\rho}_m(t)\,\delta_m(t) = 0\,, \tag{2}$$

where $H(t) \equiv \dot{a}(t)/a(t)$ is the Hubble parameter and G is the Newton gravitational constant.

To solve equation (2), it is necessary to assume a cosmological model through the Hubble parameter. In this work we solve this equation for the ΛCDM and $\omega_0\omega_a$CDM models, where the Hubble parameter is, respectively,

$$H(a) = H_0\sqrt{\Omega_{m0}a^{-3} + \Omega_{\Lambda 0}}\,, \tag{3}$$

Table 1. Cosmological parameter values assumed for the $\omega_0\omega_a$CDM and ΛCDM models. The values in the 2nd column were taken from (Planck 2018), while the data in the 3rd and 4th columns come from (DESI 2025).

Parameters\Model	ΛCDM$^{\rm Planck}$	ΛCDM$^{\rm DESI}$	$\omega_0\omega_a$CDM$^{\rm DESI}$
ω_0	-1	-1	-0.752 ± 0.057
ω_a	0	0	$-0.86^{+0.23}_{-0.20}$
Ω_m	0.315 ± 0.0073	0.3027 ± 0.0036	0.3191 ± 0.0056

Table 2. Compilation of 15 $\sigma_8(z)$ measurements.

z	$\sigma_8(z)$	error
0.013	0.78	0.04
0.24	0.67	0.04
0.47	0.58	0.04
0.53	0.59	0.03
0.60	0.59	0.02
0.63	0.53	0.04
0.69	0.66	0.10
0.80	0.47	0.04
0.83	0.58	0.04
0.92	0.44	0.06
1.10	0.48	0.01
1.50	0.46	0.05
1.59	0.39	0.06
2.72	0.22	0.06
3.80	0.12	0.06

and

$$H(a) = H_0\sqrt{\Omega_{m0}\, a^{-3} + \Omega_{\Lambda 0}\, a^{-3(1+\omega_0+\omega_a)}\, e^{-3\omega_a(1-a)}}\,, \tag{4}$$

and where Ω_{m0} and $\Omega_{\Lambda 0}$ are the density parameters of matter and dark energy today, respectively, and the scale factor a is related to the redshift z by $a = 1/(1+z)$.

The matter fluctuations amplitude at the scale $R = 8$ Mpc$/h$, denoted by $\sigma_8(z)$, is written as (Nesseris 2017)

$$\sigma_8(z) = \sigma_{8,0}\,\frac{\delta(z)}{\delta(0)}\,. \tag{5}$$

Therefore, given a cosmological model, one numerically solves equation (2) to obtain $\delta(z)$ and $\delta(0)$, then uses the equation (5) to compute the function $\sigma_8^{\rm mod}(z)$. In Table 1, we display the values assumed for the cosmological parameters for each model. For all of them, we assumed $\sigma_{8,0} = 0.8120$ from (Planck 2018).

3. σ_8 data and GP reconstruction

We consider the dataset of 14 $\sigma_8(z)$ measurements compiled by (Piccirilli 2024), plus 1 recent measurement at low-redshift by (Franco 2025a). These 15 $\sigma_8(z)$ data points were obtained analysing different cosmic tracers at redshift range $z \in [0.013,\, 3.80]$. The list of these data, with their corresponding references, is shown in Table 2. To investigate if the $\omega_0\omega_a$CDM model describes well the perturberd universe we use GP to reconstruct, in a model-independent way, the function $\sigma_8(z)$ in the interval $z \in [0,\, 4]$, function that we denote by $\sigma_8^{\rm rec}(z)$. To see more information about GP, see (Seikel 2012, 2013; Jesus 2020; Oliveira 2024).

To perform the GP reconstruction, we use the public code GaPP[1] developed by (Seikel 2012).

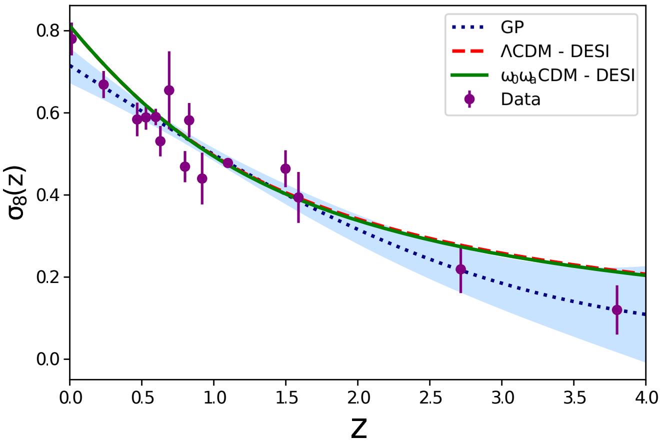

Figure 1. Gaussian Process reconstruction for the $\{\sigma_8(z_i)\}$ dataset displayed in Table 2. The dotted function represents the GP reconstruction function, $\sigma_8^{\rm rec}(z)$, with their respective $2\,\sigma$ uncertainty represented by the light-blue shadow. The dashed line in red and the continuous line in green represent respectively ΛCDM and $\omega_0\omega_a$CDM with DESI parameters.

Table 3. $\chi^2_{\rm mod/rec}$ value for each model analysed.

Model	$\chi^2_{\rm mod/rec}$
ΛCDM$^{\rm Planck}$	3.475
ΛCDM$^{\rm DESI}$	3.784
$\omega_0\omega_a$CDM$^{\rm DESI}$	3.257

4. Results and Conclusions

According to the analyses described in the previous section we have obtained the GP reconstructed function $\sigma_8^{\rm rec}(z)$ in the interval $z \in [0,\, 4]$, which is shown as a dotted curve in Figure 1. In this figure we also observe the behaviour of the models $\omega_0\omega_a$CDM and ΛCDM, both obtained with DESI best-fit parameters. We notice that the cosmological models analysed by DESI are competitive in reproducing our GP reconstructed function.

We perform a statistical comparison between $\sigma_8^{\rm mod}(z)$ from a cosmological model in study with respect to the reconstructed function $\sigma_8^{\rm rec}(z)$. For this, we define

$$\chi^2_{\rm mod/rec} \equiv \frac{1}{N}\sum_{i=1}^{N}\frac{\left[\sigma_8^{\rm mod}(z_i) - \sigma_8^{\rm rec}(z_i)\right]^2}{\sigma^{\rm mod}_{\sigma_8}(z_i)^2 + \sigma^{\rm rec}_{\sigma_8}(z_i)^2}\,, \tag{6}$$

where **mod** and **rec** refer to the cosmological model in study and to the GP reconstructed function, respectively. In these analyses we adopted $N = 1000$ bins[2]. Lower values for $\chi^2_{\rm mod/rec}$ indicate a better agreement with the data.

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[1] `https://github.com/JCGoran/GaPP`

[2] N is the number of bins used in the GP reconstruction. For consistency, we have verified that the final result is independent of N, for $N > 100$.

Using equation (6), we quantify the best-fit analysis of three cosmological models to describe the data from the clumpy universe, and the results of this statistical comparison are summarized in Table 3. In fact, regarding the best-fitting of the $\sigma_8^{\rm rec}(z)$ function, our analyses show that the model $\omega_0\omega_a$CDM$^{\rm DESI}$ is preferred over the ΛCDM$^{\rm Planck}$ and ΛCDM$^{\rm DESI}$ models. Therefore, the answer to the question in the title is that, from the three models analysed, the $\omega_0\omega_a$CDM$^{\rm DESI}$ is indeed a competitive model to reproduce the $\{\sigma_8(z_i)\}$ data from the clumpy universe.

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