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Modeling white dwarfs with strange matter cores:

A comparative analysis between different equations of state

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Abstract. This work investigates how different equations of state (EoS) affect the structure of white dwarfs by comparing standard electron-degenerate matter models with hybrid configurations containing a self-bound strange-matter core. Motivated by observational evidence of white dwarfs exhibiting radii smaller than those predicted by standard models, we explore whether ultradense central regions could be associated with strange quark matter. The stellar crust is modeled with the Baym–Pethick–Sutherland (BPS) equation of state, while the core is described by the MIT Bag Model. We solve the Tolman–Oppenheimer–Volkoff (TOV) equations numerically to construct mass–radius sequences for both conventional and hybrid stars. Our results show that, for a given mass, white dwarfs with a strange-matter core reach smaller radii and higher compactness compared to their conventional counterparts.

Resumo. Este trabalho investiga como diferentes equações de estado (EoS) afetam a estrutura de anãs brancas, comparando modelos padrões de matéria degenerada de elétrons com configurações híbridas contendo um núcleo autoligado de matéria estranha. Motivados por observações de anãs brancas cujos raios aparentam ser menores do que aqueles previstos pelos modelos tradicionais, exploramos se regiões centrais ultradensas poderiam estar associadas à matéria de quarks estranha. A crosta estelar é modelada com a equação de estado de Baym–Pethick–Sutherland (BPS), enquanto o núcleo é descrito pelo Modelo de Saco do MIT (MIT Bag Model). Resolvemos numericamente as equações de Tolman–Oppenheimer–Volkoff (TOV) para construir sequências massa–raio tanto para estrelas convencionais quanto híbridas. Nossos resultados mostram que, para uma dada massa, anãs brancas com um núcleo de matéria estranha alcançam raios menores e maior compacidade em comparação com suas contrapartes convencionais.

Keywords. Equation of state – White dwarfs – Dense matter

1. Introduction

White dwarfs (WDs) are extremely dense stellar remnants whose stability no longer depends on nuclear reactions, but rather on the quantum pressure exerted by degenerate electrons. Observationally, increasingly precise measurements of masses and radii have revealed objects whose radii appear smaller than those predicted by standard WD models (Xu 2005). This discrepancy suggests the existence of central regions denser than those of ordinary WDs, potentially approaching or exceeding nuclear saturation density, raising the possibility of a core composed of strange quark matter (Farhi & Jaffe 1984). Strange dwarfs are theoretical candidates containing a core of quark matter (u, d, s), extremely dense and self-bound.

In this study, we analyze the influence of different equations of state (EoS) on the structure of white dwarfs by numerically integrating the Tolman–Oppenheimer–Volkoff equations, considering both conventional models supported by degenerate electrons and hybrid configurations containing a self-bound strange-matter core. Our goal is to obtain the resulting mass–radius relations, allowing us to evaluate the effect of strange matter on stellar compactness and determine whether the presence of a strange core is consistent with the exceptionally small radii observed in some systems.

2. White Dwarfs

The pressure in WDs no longer originates from nuclear fusion, as in main-sequence stars. These stars are in a cooling phase, and their stability is supported by the fundamental quantum effect of electron degeneracy pressure. The energy states are filled up to a maximum value known as the Fermi energy ϵ_F, and the system is described by the Fermi–Dirac distribution in the limit $T \to 0$ (Maciel 1999).

The compulsory filling of these states determines the maximum occupation in momentum space; thus, the electron density inside the star is given by $n_e = \frac{8\pi}{3h^3} p_F^3$, and the Fermi momentum is $p_F = \hbar \left(\frac{3\pi^2 \rho Z}{m_N A} \right)^{1/3}$, where ρ is the mass density, Z and A are the atomic and mass numbers, respectively, and m_N is the nucleon mass. From the Fermi momentum, the pressure of the degenerate electron gas is obtained:

$$P = \frac{\epsilon_0}{24} \left[(2x^3 - 3x)\sqrt{1 + x^2} + 3 \ln(x + \sqrt{1 + x^2}) \right] \tag{1}$$

where $x = p_F/(m_e c)$ and $\epsilon_0 = m_e^4 c^5/(\pi^2 \hbar^3)$. This expression is the exact zero-temperature equation of state for a relativistic degenerate electron gas, providing an excellent approximation for the interior of cold white dwarfs when Coulomb and finite-temperature corrections are neglected. However, it can be simplified into a polytropic form:$P = K\rho^\gamma, \quad \gamma = 1 + \frac{1}{n}$, where the polytropic index n depends on the degeneracy regime: $n = 3/2$ in the non-relativistic case and $n = 3$ in the ultra-relativistic limit.

3. Strange Dwarfs

Strange dwarfs are composed of a core of up, down, and strange quark matter. There is a hypothesis that the true ground state of strongly interacting matter is composed of deconfined quarks. At extremely high densities, nucleons are compressed to separations smaller than the typical range of the nuclear force. In this regime, the QCD coupling constant decreases and quarks become deconfined, behaving as nearly free fermions due to asymptotic freedom. In compact stars, this deconfined phase corresponds to cold, dense quark matter, which can be modeled as a Fermi liquid of u, d, and s quarks.

With the decay of down quarks into strange quarks, three Fermi fluids can coexist and minimize the degeneracy energy, forming a self-bound state. In this context, hadronic confinement would correspond only to a metastable state (Glendenning 2012).

3.1. EoS: Bag Model

The Bag Model is a phenomenological formulation that describes quark confinement inside a finite spatial region (the bag). Quarks move freely inside the bag but cannot cross its surface due to boundary conditions imposed on the Lagrangian (Chodos et al. 1974). The effective Lagrangian is

$$\mathcal{L} = \left[\sum_f \bar{\psi}_f \left(i\gamma^\mu \partial_\mu - m_f \right) \psi_f - B \right] \theta_{\text{bag}} - \frac{1}{2} \sum_f \bar{\psi}_f \psi_f \, \delta_{\text{bag}}, \quad (2)$$

Here, ψ_f represents the quark field of flavor f. The step function θ_{bag} restricts the quarks to the interior of the bag, while δ_{bag} enforces the boundary condition at the surface. The parameter B is the bag constant, representing the QCD vacuum pressure responsible for confinement. From the Lagrangian in Eq. 2, one obtains the thermodynamic relations that lead to the equation of state:

$$P = \sum_f P_f - B, \qquad \varepsilon = \sum_f \epsilon_f + B. \quad (3)$$

In the ultrarelativistic limit ($m_f \approx 0$), one obtains the simplified relation (Farhi & Jaffe 1984; Chodos et al. 1974):

$$P = \frac{1}{3} (\epsilon - 4B) \,. \quad (4)$$

4. Model and Results

The stellar crust is modeled with the BPS equation of state, appropriate for densities between typical white dwarf envelopes and neutron drip. The crust–core transition is defined by enforcing pressure continuity at $r = R_{\text{core}}$, where a strong electric dipole layer forms due to the positively charged strange core and the surrounding electrons. At this interface, the crust density is taken just below neutron drip, following the standard strange-dwarf scenario (Glendenning 2012; Mathews et al. 2006). The core follows the Bag Model EoS, Eq. 4. Figure 1 illustrates the resulting two-layer structure, including the energy-density gap between crust and core.

The integration of the TOV equations, Eq. 5, was performed by scanning the central conditions to obtain families of $M(R)$ solutions for two scenarios: (i) conventional white dwarfs and (ii) hybrid white dwarfs with a strange-matter core. The direct comparison between these regimes is presented in Fig. 2, which shows the mass–radius curves for both models.

$$\frac{dP}{dr} = -\frac{G\,(\epsilon + P)\left(M + 4\pi r^3 P\right)}{r\left(r - 2GM\right)}, \qquad \frac{dM}{dr} = 4\pi r^2 \epsilon \quad (5)$$

The presence of a strange-matter core significantly modifies the mass–radius relation of white dwarfs. For high central densities, hybrid solutions exhibit higher compactness compared to conventional WDs. These results suggest that further adjustments to Bag Model parameters, such as the constant B and effective quark masses, may refine the physical description of these objects.

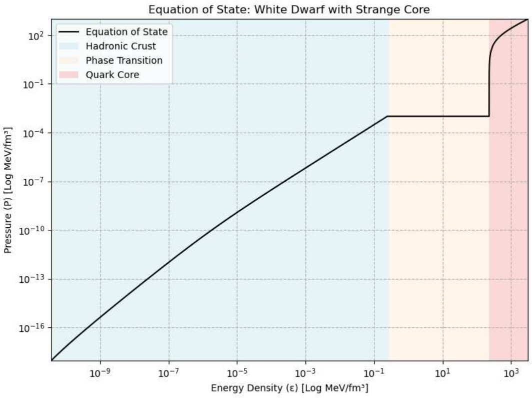

Figure 1. Pressure as a function of energy density (EoS BPS + Bag Model, considering $B^{1/4} = 145$ MeV).

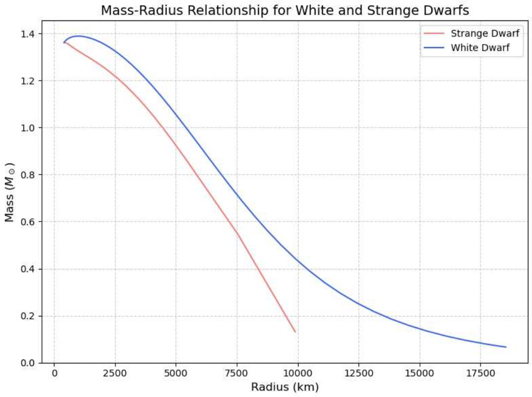

Figure 2. $M - R$ curve obtained from the integration of the TOV equations for the conventional case and the hybrid case.

5. Conclusions

The results show that strange-matter cores can significantly modify the mass–radius relation and compactness of WDs. In particular, hybrid configurations with a strange core systematically shift the $M - R$ curve toward smaller radii for a given mass, and may thus provide a viable explanation for white dwarfs whose radii appear smaller than predicted by conventional models, depending on a detailed comparison with current observational uncertainties. The structural differences between standard and hybrid models reinforce the importance of the choice of EoS in modeling these compact objects.

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