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The M–Y$_{SZ}$ scaling relation in the CODEX galaxy cluster sample

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Abstract. The aim of this work is to analyze the correlation between the Compton parameter Y_{SZ} and the mass of galaxy clusters. To this end, mass and Y_{SZ} measurements for clusters of the CODEX sample were used. The ultimate analysis of the scaling relation will be performed based on a Bayesian statistical model. The preliminary results ($\beta = 0.91$, $\gamma \neq 0$) diverge from those predicted by the self-similar model ($\beta = 1.67$, $\gamma = 0$), which represents evidence for the need to consider the temporal evolution of the scaling relation.

Resumo. O objetivo deste trabalho é analisar a correlação entre o parâmetro de Compton Y_{SZ} e a massa de aglomerados de galáxias. Para isso, foram utilizadas medições de massas e de Y_{SZ} para aglomerados da mostra *CODEX*. A principal análise da relação de escala será realizada com base num modelo estatístico Bayesiano. Os resultados preliminares ($\beta = 0.91$, $\gamma \neq 0$) divergem do previsto pelo modelo auto-similar ($\beta = 1.67$, $\gamma = 0$), o que representa uma evidência para a necessidade de se considerar a evolução temporal da relação de escala.

Keywords. Cosmology: miscellaneous – Cosmic background radiation – Large-scale structure of Universe

1. Introduction

1.1. Structure Formation and Galaxy Clusters

The Universe is structured on large scales by galaxies, groups, clusters, superclusters, and vast low-density regions. The dominant cosmological model, ΛCDM, posits these structures form hierarchically, from small "fundamental building blocks" ($\sim 10^{6} M_{\odot}$) that progressively merge over time.

Clusters are the most massive (up to $10^{15} M_{\odot}$), rarest, and dynamically mature structures. They are placed at the intersection of cosmology and astrophysics, serving as key probes for both the "dark sector" (dark matter and dark energy) and complex baryonic physics. Clusters are dominated by a hot, diffuse plasma—the Intracluster Medium (ICM)—which emits X-rays via thermal bremsstrahlung. The hot electrons in the ICM scatter Cosmic Microwave Background photons, in an inverse Compton scattering, creating the Sunyaev-Zeldovich (SZ) effect —a distinct spectral distortion observable at microwave/mm wavelengths (Sunyaev & Zeldovich (1970)). The intensity variation in the low-frequency region (Rayleigh-Jeans) can be parameterized as $\Delta I_{\nu}/I_{\nu} = -2y$. The quantity associated with the observation of the SZ effect is Y, the integral of the y parameter over the plane of the sky, which is equal to $Y = M_{g}T$, where M_{g} and T are the mass and temperature of the gas in the ICM, respectively.

1.2. Scaling Relations and the Self-Similar Model

A powerful framework for understanding clusters is the self-similar model proposed by Kaiser (1986), a purely gravitational theory predicting simple power-law (scaling) relations between cluster properties (e.g., mass, X-ray luminosity, temperature, SZ signal). Under assumptions of hydrostatic equilibrium and scale-free collapse, it predicts, for example:

- **Mass–Temperature:** $T \propto M^{2/3}$
- **Mass–SZ Signal:** $Y_{SZ} \propto M^{5/3}E(z)^{2/3}$

where $E(z) \equiv H(z)/H_0$ describes the evolution with redshift.

Observations show, however, systematic deviations from these simple predictions. Processes like radiative cooling, star formation, and energetic feedback from Active Galactic Nuclei (AGN) inject non-gravitational energy into the ICM, breaking the self-similar assumptions (Nagai (2006), Pratt (2009)). These effects alter the thermodynamic state of the gas and the scaling relation slopes.

The M — Y$_{SZ}$ correlation is most commonly worked out in logarithmic space as a linear relationship, with the slope and intercept parameters β and α, respectively, and the parameter γ for the time evolution parameterized by the redshift:

$$\log[Y_{SZ}E(z)^{-1}] = \alpha + \beta \log[M_{\Delta}E(z)] + \gamma \log(1+z). \tag{1}$$

The self-similar model predicts $\beta = 5/3$ and does not take into account the temporal evolution, that is, $\gamma = 0$.

2. Methodology

Along with Y$_{SZ}$ measurements from the CODEX (COnstrain Dark Energy X-ray clusters) catalog, obtained by the CARMA (Combined Array for Research in Millimeter-Wave Astronomy) antenna, the Planck satellite and the ACT (Atacama Cosmology Telescope) radio telescope, cluster mass measurements determined by weak lensing by Kaiser (1986), Herbonnet et al. (2020), and Oguri et al. (2021) are being used. The total sample consists of 49 clusters. The ultimate analysis of the M–Y$_{SZ}$ relation will be performed based on the Bayesian statistical model developed by Pederneiras et al. (2025), which incorporates the complex CODEX selection function and considers a series of possible systematic effects that may influence the results.

The model takes into account the probability density function of the observables, the complex CODEX selection function which includes joint detection by X-rays and optics, the mass function of the clusters, and the sampled Universe volume. Therefore, the result obtained is not specific to the sample but generalizable to the entire population of clusters. Effects such as Malmquist and Eddington biases are also fully addressed.

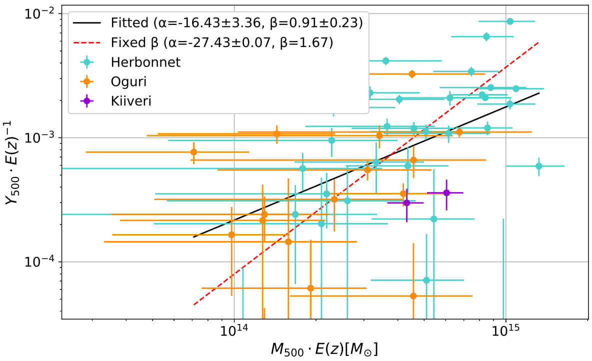

Figure 1. The M – Y_{SZ} relation in logarithmic space. The black line represents the best fit obtained for a simple linear fit in Python. The dashed red line represents the expected fit for the self-similar model.

Table 1. Free parameters α and β found by linear fitting of the M–Y_{SZ} relation.

Parameter	Value
α	-16.43 ± 3.36
β	0.91 ± 0.23

3. Results

Up to this point, a linear fit based on the least squares method has been performed to analyze the M – Y_{SZ} relation presented in Eq. 1 (Fig. 1), disregarding the time evolution term, which was evaluated separately by comparing residual and redshift fits for models with/without redshift evolution (Fig. 2) based on the Bayesian Information Criterion.

The values found for the free parameters α and β are presented in the following Tab. 1.

A z-test performed between the obtained and expected values of β results in $z = \frac{x-\mu}{\sigma} = \frac{1.67-0.91}{0.23} = 3.3$. That is, considering a significance level of 3σ, our preliminary result is incompatible with that predicted by the self-similar model.

The temporal evolution of $M - Y_{SZ}$ was analyzed separately from a linear fit of the residuals and the redshift, which was compared with a second fit with a fixed slope parameter, as illustrated in Fig. 2. This latter fit has only one free parameter (instead of two) and corresponds to the model in which there is no evidence of evolution.

To compare the two fits, their Bayesian Information Criterion (BIC) values were used to calculate $\Delta BIC = BIC_{model} - BIC_{best}$, which is the difference between a model BIC_{model} and the best model BIC_{best} (i.e., the one with the lowest BIC value). The BIC parameter seeks to maximize the posterior of the fit, introducing penalty terms for the number of parameters in the model, effectively mitigating the risk of overfitting. The value of ΔBIC can be considered a test of the evidence against the new model (Arevalo (2017)), such that:

- For $\Delta BIC < 2$, the evidence against the new model is considered to be worth mentioning.
- For $2 < \Delta BIC < 6$, the evidence against the new model is considered to be positive.
- For $6 < \Delta BIC < 10$, the evidence against the new model is considered to be strong.
- For $\Delta BIC > 10$, the evidence against the new model is considered to be very strong.

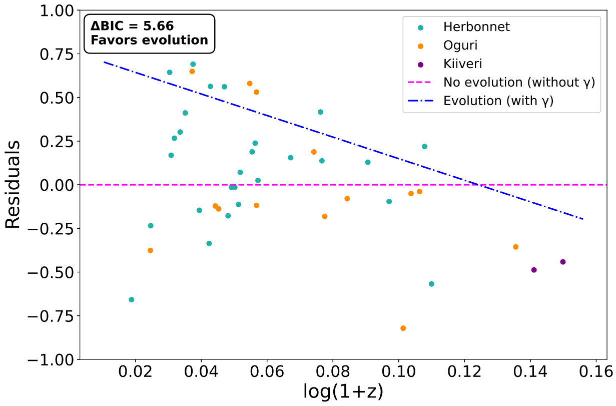

Figure 2. Residuals of the linear fit as a function of redshift. The pink dashed line represents the best fit for a model without temporal evolution, and the blue line for a model with temporal evolution.

The values obtained were $BIC_{best} = -72.34$ for the first fit (with two free parameters and corresponding to a model with evolution) and $BIC_{model} = -66.68$ for the second fit (with only one parameter and corresponding to the model where there is no evidence of evolution). Thus, $\Delta BIC = 5.66$, which indicates positive evidence in favor of the model with evolution.

4. Conclusions

From the results presented above, it is clear that the parameters of the M–Y_{SZ} relation found ($\beta = 0.91$, $\gamma \neq 0$) diverge from that predicted by the self-similar model ($\beta = 1.67$, $\gamma = 0$), which represents evidence for the need to consider the temporal evolution of the scaling relation and the various thermodynamic transformations undergone by the clusters.

For the result to have physical meaning, however, we still need to consider a number of systematic errors in our model, such as the Malmquist and the Eddington biases. We intend to improve the quality of our analysis by applying the Bayesian statistical model discussed in Sec.2, which accounts for several systematic errors and selection functions that may influence our results.

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