

Viscosity effects on the common envelope phase of binary systems

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Abstract. The observed separations between stars in binary systems require the existence of a common envelope (CE) phase to bring the stars closer together. First proposed by Paczyński (1976), this phase occurs when the envelope of a giant star engulfs its companion, forming a shared envelope in which energy and momentum are exchanged. However, this stellar phase is not well understood, partly due to the lack of observational data on CEs, but also because of theoretical uncertainties. The energetics of the system is often invoked to explain envelope ejection, yet the key physical processes operating during this phase remain unknown. Moreover, studies of CE energetics frequently overlook the contribution of turbulent kinetic energy to the system's dynamics. In this work, we propose that turbulent kinetic energy and viscous dissipation play a central role in the redistribution of energy within the envelope. By combining the Kolmogorov spectrum with the Reynolds-Averaged Navier–Stokes equations, and adopting a prescription for turbulent viscosity, we formulate a model that incorporates turbulence into the CE energy balance. This approach enables the calculation of turbulent kinetic energy and is expected to lead to a new formulation for common envelope evolution, which is currently under investigation.

Resumo. As separações observadas entre estrelas em sistemas binários exigem a existência de uma fase de envelope comum (CE) para aproximar os componentes estelares. Proposta inicialmente por Paczyński (1976), essa fase ocorre quando o envelope de uma estrela gigante envolve sua companheira, formando um envelope compartilhado no qual energia e momento são trocados. No entanto, essa fase evolutiva não é bem compreendida, em parte devido à escassez de dados observacionais sobre envelopes comuns mas também pelas incertezas na modelagem teórica. A análise energética do sistema é frequentemente utilizada para explicar a ejeção do envelope, mas os processos físicos fundamentais que operam durante essa fase permanecem desconhecidos. Além disso, estudos sobre a energética de CEs frequentemente negligenciam a contribuição da energia cinética turbulenta para a dinâmica do sistema. Neste trabalho, propomos que a energia cinética turbulenta e a dissipação viscosa desempenham um papel central na redistribuição de energia dentro do envelope. Ao combinar o espectro de Kolmogorov com as equações de Navier–Stokes e adotar uma prescrição para a viscosidade turbulenta, iniciamos a formulação de um modelo que incorpora a turbulência no balanço energético do CE. Essa abordagem permite calcular a energia cinética turbulenta e é potencialmente capaz de levar a uma nova formulação para a evolução de envelopes comuns, atualmente em investigação.

Keywords. binaries: close – turbulence – stars: evolution

1. Introduction

The majority of stars in binary and multiple systems are closely bound. Observations of these systems at the end of their lives reveal that the distances between remnants are orders of magnitude smaller than simple predictions from population synthesis studies. Therefore, revisions to the possible remnant formation channels are required. To address this discrepancy, it has been proposed that a common envelope (CE) phase exists (Paczynski 1976), in which the envelope of a giant star engulfs its companion and they exchange energy and angular momentum.

If the stars survive the CE phase, a binary system with a tighter orbit is formed, otherwise a merger may occur. This phase can be present in the formation channels of double neutron star systems, black holes, white dwarfs, type Ia supernovae, and X-ray binaries, but it introduces uncertainties and free parameters because the main physical processes involved in common envelope evolution (CEE) are not well understood and there is no consensus regarding observational evidence of the phase's occurrence (Ivanova et al. 2013).

To describe the CEE, two main approaches are commonly used: the α -formalism, based on the idea that changes in the system's orbital energy drive orbit tightening and envelope ejection (van den Heuvel 1976; Livio & Soker 1988), and the γ -formalism, which attributes these effects to the loss of angular momentum from the binary systems (Nelemans et al. 2000).

Nevertheless, orbital energy alone or conservation of angular momentum have not been capable of reproducing the observed

post-CE separations consistently across different types of systems (Ivanova et al. 2013). Even though other energy sources such as internal and recombination (Han et al. 1994), nuclear fusion (Ivanova & Podsiadlowski 2003), dust winds (Glanz & Perets 2018), jets (Soker 2017), and radiation pressure (Chen 2025) have been suggested recently, there is still no consensus on any of them being capable of reproducing the small separations required between binaries under the common envelope channel hypothesis (Ivanova et al. 2013), especially when energy transport is considered (Soker & Harpaz 2003; Sabach et al. 2017; Grichener et al. 2018; Wilson & Nordhaus 2022).

Because frictional drag is caused by the spiral-in movement of the stars inside the envelope, it removes energy and momentum from the orbit (Paczynski 1976). However, works based on the energy formalism generally neglect the processes of energy transport and dissipation in their calculations (Wilson & Nordhaus 2022) and attribute only an efficiency parameter to the variations between initial and final systems (Livio & Soker 1988), which has also been shown to be inconsistent for a single explanation of the different types of systems so far. Furthermore, hydrodynamical simulations based on drag forces have been performed, but they do not consider the effects of turbulent viscosity, require substantial computational resources over different timescales, and have not been able to implement all the dynamics of the CEE (Ivanova et al. 2013; Röpke & De Marco 2023).

In this ongoing project, we have been developing an approach based on the conservation of energy that considers the turbulent viscosity of the common envelope. Using the Kolmogorov spec-

trum and the Navier–Stokes equations, we expect to quantify the impacts of turbulent kinetic energy on envelope ejection and its influence as an energy reservoir or sink.

2. The turbulent formalism

Since the CE encompasses the envelope of a giant star, and stars on the Red Giant Branch and Asymptotic Giant Branch present highly turbulent envelopes (Horvath 2011), the CE may be a turbulence-dominated medium. Consequently, the kinetic energy of the flow is dissipated as heat, and efficient energy transport can occur.

If this is the case, viscosity transforms the turbulent kinetic energy of the envelope shells into heat, which may act as either a source or a sink of energy for envelope ejection when the system’s internal energy is considered. Moreover, the friction between the stars and the envelope may extract energy from the binary system and transfer it to the CE.

When turbulence is taken into account, the resulting energy transport rate can be much higher than that associated with radiative or convective transport (Horvath 2011). Thus, depending on the CE opacity, potentially important energy sources may in fact be lost from the CE. Even with convection alone, there is evidence that energy sources such as ionization energy cannot be responsible for envelope ejection, since they would be radiated away from the envelope (Soker & Harpaz 2003; Sabach et al. 2017; Grichener et al. 2018).

Kolmogorov’s theory of turbulence assumes that, in a fully developed turbulent flow, energy is transferred from large eddies to progressively smaller scales, until reaching the smallest scale in the inertial subrange, where viscosity dissipates it as heat (Kolmogorov 1941a). Assuming that this theory can be applied to the CEE, the energy of the large turbulent eddies in the envelope follows this cascade until dissipation occurs. This energy cascade follows a power-law, with the energy spectrum $E(k)$ decreasing as (Kolmogorov 1941b):

$$E(k) = C_D \varepsilon^{2/3} k^{-5/3} \quad (1)$$

Where C_D is the Kolmogorov’s constant, ε is the dissipation rate and k is the wave number.

In the modern description of turbulence, this cascade is sustained by the transfer of kinetic energy from the mean flow of the CE into turbulent motions. These turbulent fluctuations then redistribute the energy across scales, ultimately reaching the viscous range where it is converted into heat (Ting 2016).

On the other hand, the CE can be modeled as an incompressible flow in spherical coordinates using the continuity and Navier–Stokes equations:

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta u_\theta) + \frac{1}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} = 0 \quad (2)$$

$$\begin{aligned} \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_r}{\partial \phi} - \frac{u_\theta^2 + u_\phi^2}{r} \\ = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[\nabla^2 u_r - \frac{2u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial u_\phi}{\partial \phi} \right] \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi} + \frac{u_r u_\theta - u_\phi^2 \cot \theta}{r} \\ = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + \nu \left[\nabla^2 u_\theta - \frac{u_\theta + 2 \cot \theta u_\phi}{r^2 \sin^2 \theta} \right] \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r u_\phi + u_\theta u_\phi \cot \theta}{r} \\ = -\frac{1}{\rho r \sin \theta} \frac{\partial p}{\partial \phi} + \nu \left[\nabla^2 u_\phi - \frac{u_\phi - 2 \cot \theta u_\theta}{r^2 \sin^2 \theta} \right] \end{aligned} \quad (5)$$

The Kolmogorov spectrum can be combined with these equations within the Reynolds–Averaged Navier–Stokes framework and applied to a spherically symmetric CE. A model for the turbulent viscosity is also required. With reasonable simplifications of these equations, it should become possible to calculate the turbulent kinetic energy and obtain a new condition for the common envelope evolution and ejection, a work currently in progress.

3. Conclusions and future work

Close binaries require an evolutionary channel commonly associated with the common envelope phase. However, there is no established mechanism for envelope ejection, and the energy sources proposed so far appear to be insufficient. Moreover, most of these mechanisms do not explicitly account for turbulence or energy transport. In our work, we propose that turbulent kinetic energy and convection may play a fundamental role in the redistribution and dissipation of energy within the envelope. In this context, the Kolmogorov spectrum provides a useful way to describe the cascade of turbulent energy and its eventual dissipation as heat. Future work will focus on incorporating viscosity effects into numerical simulations based on the Navier–Stokes equations, and on comparing the resulting theoretical predictions with observational constraints from compact binary systems.

Acknowledgements. We would like to thank FAPESP for supporting this research, 2025/01641-7.

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