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Angular momentum loss of low–mass stars

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Abstract. A traditional model used to describe the evolution of the angular velocity, Ω, of low–mass stars considers that Ω remains constant over a time $\tau_{\rm disk}$, while the star is magnetically locked to its disk of gas and dust. As a consequence of this interaction, angular momentum is transferred from the star to the disk. When the system reaches an age equal to $\tau_{\rm disk}$, the decoupling between the star and the disk occurs, allowing the variation of the angular velocity. In the following stages of the angular momentum evolution, Ω increases due to the star contraction. For a solar mass star, the angular velocity reaches a maximum value in approximately 30 Ma and, after that, the contraction practically stops and Ω decreases due to angular momentum loss caused by stellar winds. Some works have proposed a correlation between the angular momentum loss and the Rossby number, *Ro*, defined as the ratio between the rotation period and the convective turnover time. Based on this, we aim to investigate a possible relationship between the disk lifetime and the Rossby number. Exploring different values of initial rotation period and disk lifetime with a Markov Chain Monte Carlo (MCMC) method, we model the evolution of the angular velocity for low-mass stars, using stellar parameters provided by the ATON stellar evolution code. We define the critical Rossby number, $Ro_{\rm crit}$, as the value of *Ro* when the star is no longer locked to its disk. Our results suggest a decay of $Ro_{\rm crit}$ as a function of disk lifetime and a correlation with stellar mass in the mass range investigated in the model.

Resumo. Um modelo tradicional utilizado para descrever a evolução da velocidade angular, Ω, de estrelas de baixa massa considera que Ω permanece constante durante um tempo $\tau_{\rm disk}$, enquanto a estrela encontra-se ligada magneticamente ao seu disco de gás e poeira. Através dessa interação, momento angular é transferido da estrela para o disco. Quando a idade do sistema iguala-se a $\tau_{\rm disk}$, ocorre o destravamento entre a estrela e o disco, permitindo a variação da velocidade angular. Nos estágios seguintes da evolução do momento angular, ocorre um aumento de Ω devido à contração da estrela. Para estrelas de massa solar, a velocidade angular atinge um valor máximo em aproximadamente 30 Ma e, após isso, a contração praticamente pára e ocorre um decréssimo de Ω devido à perda de momento angular através de ventos estelares. Alguns trabalhos sugerem que a perda de momento angular está relacionada ao número de Rossby, *Ro*, definido como a razão entre o período de rotação e o tempo convectivo local. Baseados nisso, investigamos uma possível relação entre o tempo de disco e o número de Rossby. Explorando diferentes valores de período de rotação inicial e tempo de disco através do método de Monte Carlo via Cadeias de Markov (sigla em inglês, MCMC), modelamos a evolução da velocidade angular de estrelas de baixa massa, utilizando parâmetros estelares fornecidos pelo código de evolução estelar ATON. Denominamos número de Rossby crítico, $Ro_{\rm crit}$, o valor de *Ro* quando a estrela se desprende do disco. Os nossos resultados sugerem um decréssimo de $Ro_{\rm crit}$ em função do tempo de disco e uma correlação com a massa estelar dentro do intervalo de massas usado no modelo.

Keywords. Stars: pre-main sequence – Stars: rotation – Stars: low-mass – Stars: evolution – (Stars:) circumstellar matter

1. Introduction

The study of the stellar angular velocity and angular momentum evolution is fundamental to understanding the physical processes that occur in stellar interiors related to magnetic activity. One of the first works to present a relation between the stellar angular velocity and magnetic activity was that of Skumanich (1972). For ages higher than approximately 40 million years, he found that the angular velocity at the stellar equator decays as the $\tau^{-0.51}$, where τ is the age of the star in Giga years. This relationship between the angular velocity and age is called the Skumanich law.

In the context of star formation, it is believed that the passage of shock waves can cause disturbances in stellar clouds, resulting in the birth of stars in dense fragments of these clouds. A circumstellar disk is formed due to the conservation of angular momentum, as the core of the rotating gas fragment contracts. In the early stages of the star life, the magnetosphere of the protostar is coupled to its disk of gas in a process called disk-locking.

While the protostar remains magnetically connected to the disk of gas and dust (condition that lasts for a time $\tau_{\rm disk}$), its rotational velocity is constant, equal to the initial value of the angular velocity, $\Omega = \Omega_0$. After this interaction ceases, the angular velocity can vary. The angular momentum is defined as $J = I\Omega$, where I is the momentum of inertia, and the variation of the angular velocity can be written as:

$$\frac{d\Omega}{dt} = \frac{dJ}{dt}\frac{1}{I} - \frac{dI}{dt}\frac{\Omega}{I}. \tag{1}$$

As demonstrated by Kawaler (1988), the variation of the angular momentum can be expressed as follows:

$$\frac{dJ}{dt} = -K_{\rm W}\Omega^{1+(4an/3)}\left(\frac{R}{\rm R_\odot}\right)^{2-n}\left(\frac{\dot{M}}{-10^{-14}{\rm M_\odot/yr^{-1}}}\right)^{1-2n/3}\left(\frac{M}{\rm M_\odot}\right)^{n/3}, \tag{2}$$

where $K_{\rm W}$ represents a constant, R is the radius of the star, $\dot{M}$ is the mass loss rate and the value of the parameter n depends on the magnetic field geometry, and is commonly considered equal to 1.5, an intermediary configuration between the radial and dipolar magnetic field. The constant a depends on the type of star and is typically set to 1 for late-type stars. Eq. 2 considers the loss of angular momentum exclusively due to stellar winds.

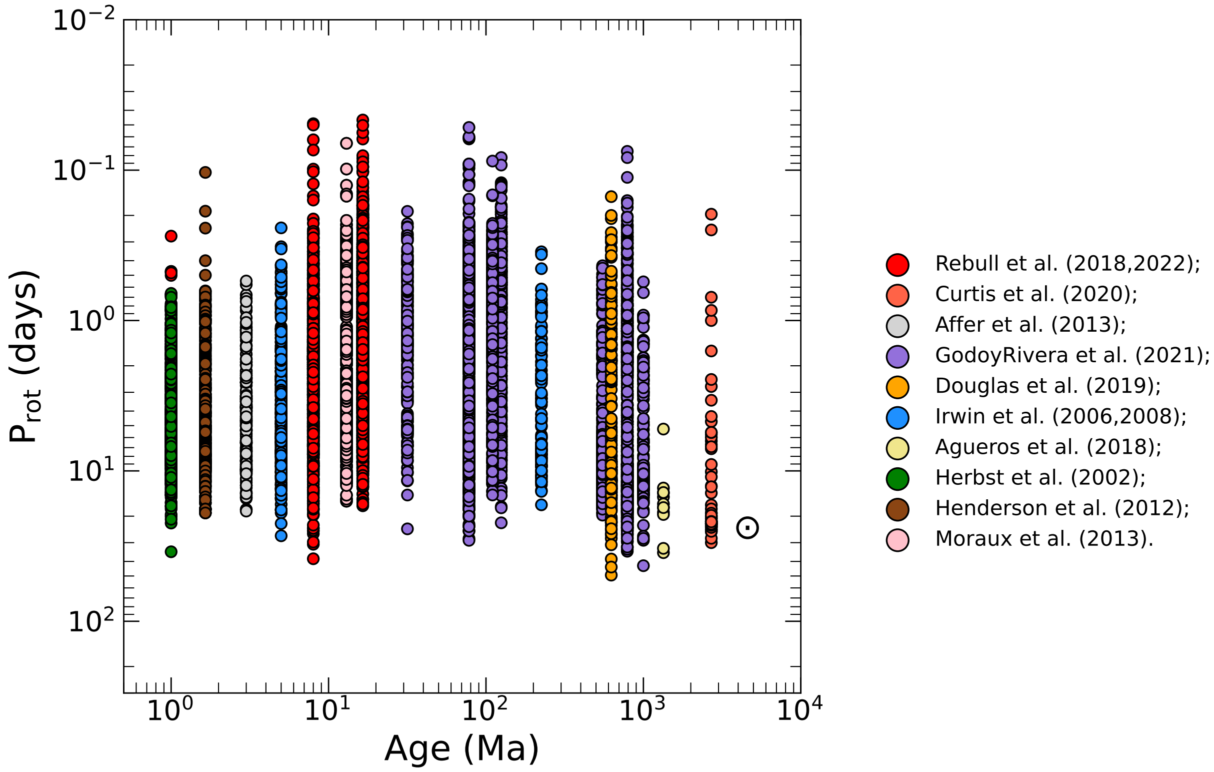

Figure 1. Rotation period as a function of age for stars from different clusters. As we consider that stars in the same cluster are exactly the same age, each dotted column represents a different cluster. We show data from the following clusters and stellar associations: UCL and LCC from Rebull et al. (2022), Ruprecht 147 from Curtis et al. (2020), NGC 2264 from Affer et al. (2013), Upper Scorpion and ρOph from Rebull et al. (2018), NGC 2547, Pleiades, M50, NGC 2516, M37, Presepio, and NGC 6811 from Godoy-Rivera, Pinsonneault, & Rebull (2021), Hyades from Douglas et al. (2019), M34 from Irwin et al. (2006), NGC 752 from Agüeros et al. (2018), ONC from Herbst et al. (2002), NGC 6530 from Henderson & Stassun (2012), NGC 2362 from Irwin et al. (2008), and h Persei from Moraux et al. (2013).

In their analysis, Bouvier, Forestini, & Allain (1997) proposed an expression in two parts, separated by the saturation of the angular velocity, for dJ/dt. They considered that, as assumed by Barnes & Sofia (1996), the increase in angular momentum loss in the saturation regime is not as intense as in the regime of low angular velocity. Allain (1998) proposed an equation in three parts for the angular momentum evolution with the aim of reproducing the Schumanich law, the saturation regime of the angular velocity, and the regime for high velocity, initially discussed by Mayor & Mermilliod (1991), where dJ/dt depends on Ω^2.

The works of Kawaler (1988) and Bouvier, Forestini, & Allain (1997) considered only angular momentum loss due to stellar winds. Just as Allain (1998), Gallet & Bouvier (2013) and Gallet & Bouvier (2015) took into account other physical processes that are responsible for changes in the angular velocity through the life of the star. These works consider the influence of the radiative core growth, which is responsible for increasing the value of the momentum of inertia in the internal radiative region, as well as transferring angular momentum between the radiative core and the convective envelope.

In this context, the evolution of Ω is calculated for the core and the envelope regions separately. In addition, Gallet & Bouvier (2013, 2015) employed a different formulation than that used in Eq. 2 for the loss of angular momentum due to stellar winds. They integrated Eq. 1 to describe the percentiles of the rotation period distribution for stars in different clusters, and it was assumed that all stars in the same cluster are exactly the same age. Gallet & Bouvier (2015) described the evolution of the angular velocity considering three different masses, 1 $M_\odot$, 0.8 $M_\odot$, and 0.5 $M_\odot$. As a result, it was found that the initial rotation period and τ_{disk} are correlated and disk lifetimes increase with mass for slow and median rotators.

van Saders et al. (2016) used a law for angular momentum loss that is analogous to Eq. 2, but divided into three parts and regulated by the Rossby number to describe dJ/dt and, consequently, the evolution of Ω. Furthermore, they used rigid body rotation. Using this new and dramatically weakened magnetic braking law, they verified the dependence of the stellar angular momentum loss regime on the Rossby number. They were also able to describe the angular velocity evolution of solar-like stars more evolved than the Sun, by explaining the existence of the unexpected rapid rotation in these old stars.

It is possible to study the stellar angular velocity evolution through other methods. By using the disk-locking mechanism, Vasconcelos & Bouvier (2015) constructed synthetic clusters and were able to reproduce the rotation period distributions of three young clusters, the Orion Nebula Cluster (ONC), NGC 2264, and h Per. Vasconcelos et al. (2022) used the core-envelope decoupling model proposed by Gallet & Bouvier (2015) to also investigate clusters with ages up to 500 Ma. They proposed that, for stars exhibiting low and median angular velocities, the coupling between the core and envelope is comparatively weaker than for fast rotators.

In another context, Landin et al. (2016) investigated the evolution of the stellar angular velocity using the stellar evolutionary code ATON, which takes into account the effects of rotation on the star's internal structure. They examined the rotation properties of two young stellar clusters, ONC and NGC 2264, and determined that the appropriate range of $\tau_{\rm disk}$ for describing moderate rotators from the ONC and NGC 2264 was $0.2 \leq \tau_{\rm disk}$ (Ma) ≤ 3 and $0.2 \leq \tau_{\rm disk}$ (Ma) ≤ 10, respectively.

In this work, we employed the formulation of dJ/dt from Allain (1998) to solve Eq. 1, using stellar parameters from the ATON code (Landin et al. 2023). As in Gallet & Bouvier (2013, 2015), the aim is to describe the percentiles of rotation period distributions from different clusters of different ages. Here, rigid body rotation and only angular momentum loss due to stellar winds were considered.

In Sec. 2, the model of the evolution of angular velocity and the evolutionary code used to obtain the stellar parameters are described. The clusters and their respective ages are discussed in Sec. 3. The analysis made to compare theoretical results with observational data is described in Sec. 4. The results and discussion are in Sec. 5. The conclusion is in Sec. 6.

2. Model

With the purpose of solving Eq. 1, we considered the angular momentum evolution model described below,

$$\frac{dJ}{dt} = \begin{cases} -K_{\rm sk}\Omega^3 \left(\frac{R}{\rm R_\odot}\right)^{1/2} \left(\frac{M}{\rm M_\odot}\right)^{-1/2} & \text{se } \Omega < \Omega_{\rm crit}, \\ -K_{\rm mm}\Omega^2 \left(\frac{R}{\rm R_\odot}\right)^{1/2} \left(\frac{M}{\rm M_\odot}\right)^{-1/2} & \text{se } \Omega_{\rm sat} > \Omega \geq \Omega_{\rm crit}, \\ -K_{\rm mm}\Omega\Omega_{\rm sat} \left(\frac{R}{\rm R_\odot}\right)^{1/2} \left(\frac{M}{\rm M_\odot}\right)^{-1/2} & \text{se } \Omega \geq \Omega_{\rm sat}, \end{cases} \quad (3)$$

and presented by Allain (1998), where M is the stellar mass, $K_{\rm sk}$ and $K_{\rm mm}$ are constants, $\Omega_{\rm sat}$ is the saturation of the angular velocity, and $\Omega_{\rm crit}$ determines the regime of rapid rotators. As in Allain (1998), we used $K_{\rm sk} = 2,7 \times 10^{47}$, $K_{\rm mm} = 4,2 \times 10^{42}$, and $\Omega_{\rm crit} = K_{\rm mm}/K_{\rm sk}$. In addition, we considered $\Omega_{\rm sat} = 30\ \Omega_\odot$.

The radius R and the momentum of inertia of a star of a given mass and age were obtained from the ATON code (Landin et al. 2023). We used non-rotating models to obtain the stellar parameters. We assumed that the convection efficiency from the Mixing Length Theory (Böhm-Vitense 1958), α, is equal to 2. The surface boundary conditions were obtained from non-gray atmosphere models by Allard, Hauschildt & Schweitzer (2000). The adjustment between the interior and the atmosphere was made at an optical depth of 3.

3. The stellar sample

We obtained some stellar parameters from the literature, like the rotation period, age, and mass for stars from the following clusters or associations: ONC with 1 Ma from Herbst et al. (2002); ρ Oph and Upper Scorpion with 1 Ma and 8 Ma, respectively, from Rebull et al. (2018); NGC 6530 with 1.65 Ma from Henderson & Stassun (2012); NGC 2264 with 3 Ma from Affer et al. (2013); NGC 2362 with 5 Ma from Irwin et al. (2008); h Persei with 13 Ma from Moraux et al. (2013); Upper Centaurus Lupus (UCL) and Lower Centaurus Crux (LCC), whose ages are equal to 16.5 Ma from Rebull et al. (2022); M34 with 225 Ma from Irwin et al. (2006); the Hyades with 625 Ma from Douglas et al. (2019); NGC 752 with 1340 Ma from Agüeros et al. (2018); NGC 2547 with 31.7 Ma, M50 with 78 Ma, NGC 2516 with 110 Ma, the Pleiades with 125 Ma, M37 with 550 Ma, Presepio with 790 Ma, and NGC 6811 with 1000 Ma from Godoy-Rivera, Pinsonneault, & Rebull (2021); Ruprecht 147 with 2700.0 Ma from Curtis et al. (2020). Fig. 1 illustrates the various data sets that were obtained in the rotation period-age plane.

4. Method

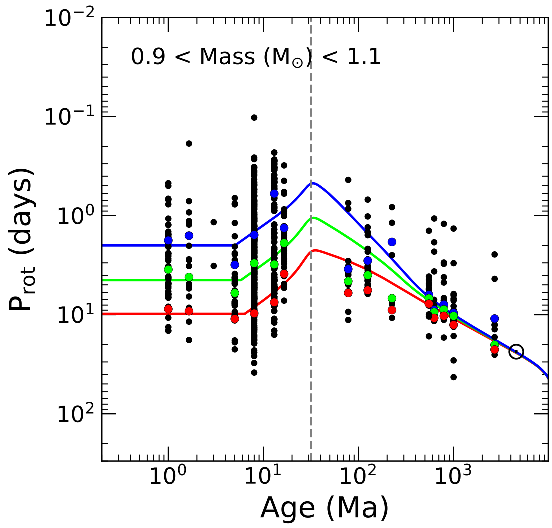

Figure 2. Rotation period as a function of age. Black dots are stars with mass between 0.9 $\rm M_\odot$ and 1.1 $\rm M_\odot$ from various clusters. The red, green, and blue dots are the percentiles of 90th, 50th, and 25th of the rotation period distributions, respectively. The curves correspond to the most likely 1 $\rm M_\odot$ models that fit the percentiles up to approximately 30 Ma, represented by the vertical dashed line.

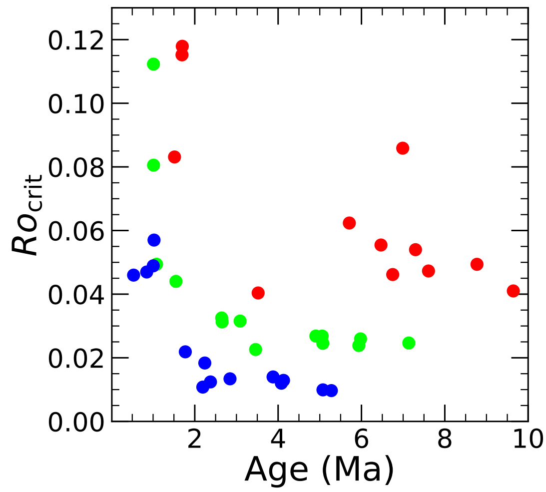

Figure 3. Critical Rossby number as a function of age. Blue, green, and red dots are $Ro_{\rm crit}$ from fast, median, and slow rotators models.

In order to investigate the dependence of angular velocity evolution on stellar mass, the data were divided into distinct mass ranges (in solar masses), distributed as follows: 0 to 0.2, 0.1 to 0.3, 0.2 to 0.4, 0.3 to 0.5, 0.4 to 0.6, 0.5 to 0.7, 0.6 to 0.8, 0.7 to 0.9, 0.8 to 1.0, 0.9 to 1.1, 1.0 to 1.2, 1.1 to 1.3, 1.2 to 1.4, and 1.3 to 1.5. In order to model the angular velocity evolution in each mass range, we used the average mass of the interval. We did not obtain a significant amount of data for $M > 1.4\ \rm M_\odot$ to conduct our analyses, and because of that, we considered only masses from 0.1 $\rm M_\odot$ to 1.4 $\rm M_\odot$.

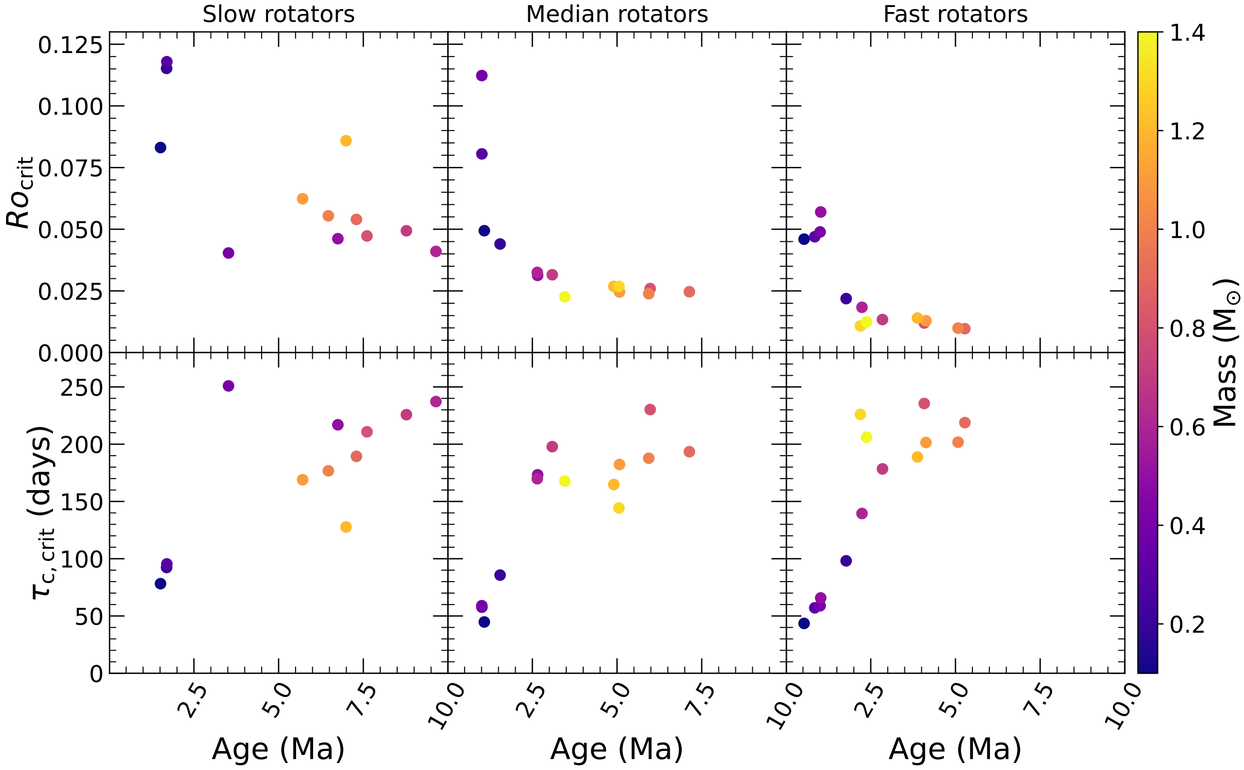

Figure 4. Critical Rossby number (top) and critical convective turnover time (bottom) as a function of age. The color map seen on the right of the figure corresponds to stellar mass. The panels, from left to right, show the results for slow, median, and fast rotators models, respectively.

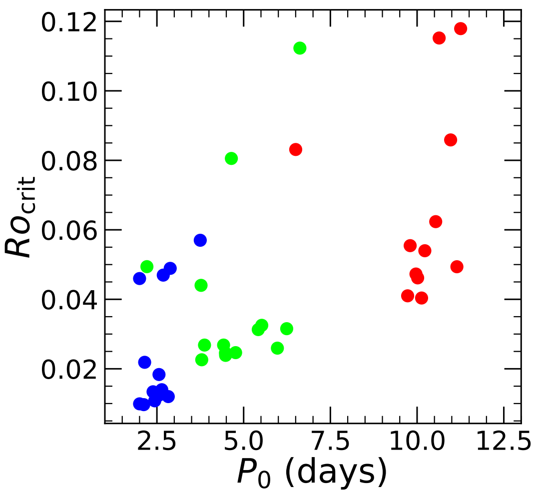

Figure 5. Critical Rossby number as a function of the initial rotation period. The colors are the same as in Fig. 3.

Subsequently, the percentiles of the rotation period distribution were calculated for each cluster, as illustrated in Fig. 2, which shows a plot of rotation period versus age for stars from 0.9 to 1.1 $M_\odot$. The 90th, 50th, and 25th percentiles represent slow, median, and fast rotators, respectively. The most probable values of the initial rotation period and disk lifetime for each mass interval were obtained through a Markov Chain Monte Carlo (MCMC) analysis (Foreman-Mackey et al. 2013).

In the fitting procedure, only percentiles defined up to 30 million years were considered. This analysis is constrained to the initial stages of angular velocity evolution, because our objective is to investigate if the coupling between the star and the disk is governed by the Rossby number, and this coupling lasts for a few million years. Considering only these early stages, it is possible to take only P_0 and τ_{disk} as free parameters, since these two parameters are those that influence most the angular velocity evolution up to 30 Ma.

The curves that describe the percentiles in the early stages of the angular velocity evolution for $M = 1\ M_\odot$ are illustrated in Fig. 2. The models describe accurately the late stages of rotation for masses from 0.6 $M_\odot$ to 1.2 $M_\odot$.

5. Results

With the purpose of obtaining the most probable disk lifetime that describes the stellar angular momentum evolution, we calculated the Rossby number corresponding to τ_{disk}, using the ATON code (Landin et al. 2023). These Rossby numbers were designated as critical Rossby numbers, denoted by Ro_{crit}. Fig. 3 presents Ro_{crit} as a function of age. It is possible to see that there is a tendency that the slower the rotation regime, the higher the values of the critical Rossby number. The data suggest that Ro_{crit} decays abruptly at an age around 2 Ma for fast, median, and slow rotators, with slow rotators displaying greater dispersion than the other rotation regimes.

In Fig. 4 is shown the critical Rossby number and the critical convective turnover time, $\tau_{c,crit}$, as a function of age for slow, median, and fast rotators. In contrast to Ro_{crit}, $\tau_{c,crit}$ varies approximately linearly with age and shows a dispersion for ages greater than 2.5 Ma. The color map highlights the dependence between these parameters and the stellar mass. As shown in Fig. 4, our results suggest that there is a tendency that the higher the stellar mass, the higher the disk lifetime, since lower-mass stars ($M \lesssim 0.5$ M$_\odot$) achieve the critical Rossby number earlier. Similarly, Gallet & Bouvier (2015) found that the $\tau_{\rm disk}$ increases with mass, except for fast rotators.

Fig. 5 depicts the critical Rossby number as a function of the initial rotation period. As expected, fast rotators exhibit smaller values of P_0. Fig. 5 shows that fast rotators also exhibit smaller values of critical Rossby numbers compared to median and slow rotators.

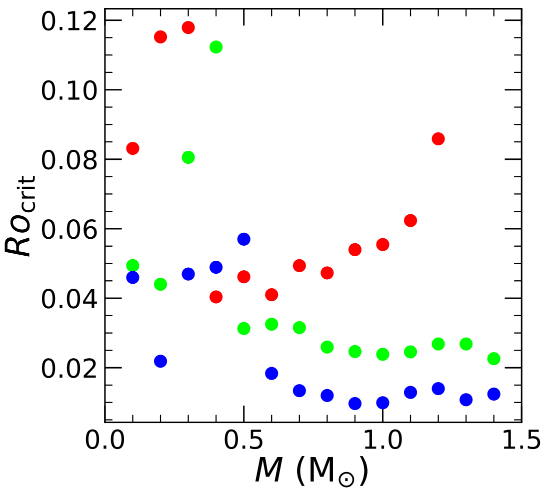

Figure 6. Critical Rossby number as a function of mass. The colors are the same as in Fig. 3.

Fig. 6 shows $Ro_{\rm crit}$ as a function of mass. For masses below 0.5 M$_\odot$, no correlation is evident in the data. For masses above 0.5 M$_\odot$, the critical Rossby number increases with mass for slow rotators. In contrast, for median and fast rotators, there is not a significant variation of $Ro_{\rm crit}$ with mass, although a slightly decrease can be observed, with median rotators exhibiting higher values of critical Rossby number than fast rotators.

6. Conclusion

The present study examined the relation between the Rossby number and the coupling between the star and its circumstellar disk of gas and dust. We compiled data of stellar rotation periods as a function of age for different clusters with ages varying from 1 Ma to 3×10^3 Ma. The slow, median, and fast rotators were defined as the 90th, 50th, and 25th percentiles from the rotation period distributions. The angular velocity variation was calculated considering the disk lifetime and the initial rotation period as free parameters in the MCMC analysis. Using the ATON stellar evolutionary code, we calculated the Rossby number corresponding to an age equal to $\tau_{\rm disk}$ and defined it as the critical Rossby number, $Ro_{\rm crit}$. Our results suggest that the higher the rotation regime, the higher $Ro_{\rm crit}$. Additionally, the critical Rossby number decays abruptly with age, for all rotation regimes. On the other hand, the critical convective turnover time shows a linear correlation with age. Finally, we found a tendency that the higher the stellar mass, the longer is the disk lifetime.

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