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Tidal dissipation on Saturn and the impact on the orbital expansion of Mimas

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Abstract. Mimas is the closest mid-sized satellite to Saturn. It currently forms a well-known dynamical system together with Enceladus, Tethys, and Dione. The tidal interaction with Saturn draws the orbital dynamics of its satellites causing them to migrate, thus altering their semi-axes and eccentricities. In addition, the tide causes them to be captured in orbital resonances. In addition to the 4 : 2 resonance with Tethys, Mimas evolves in resonance with three other small satellites in its close vicinity: Aegaeon, Methone, and Anthe. This resonant configuration brought some questions about the migration of Mimas. Here, we investigate Mimas migration considering a Maxwell rheology for the Saturn tide and creep theory for the Mimas tide. We describe Saturn as a two-layer body and analyze two possible cases: when there is only dissipation in the solid core and when there is only dissipation in the fluid envelope. We analyze the Mimas migration through the numerical solution of the averaged equations in a scenario where the dissipation factor Q is not constant, to extend our understanding of its orbital evolution in about 10^9 years.

Resumo. Mimas é o satélite mediano mais próximo de Saturno. Atualmente forma um conhecido sistema dinâmico junto com Enceladus, Tethys e Dione. A interação das marés com Saturno desenha a dinâmica orbital de seus satélites promovendo migrações e alterando assim seus semi-eixos e excentricidades. Também é possível que a maré capture os satélites em ressonâncias orbitais. Além da ressonância 4:2 com Tethys, Mimas evoluiu em ressonância com três outros pequenos satélites na sua vizinhança próxima: Aegaeon, Methone e Anthe. Essa configuração ressonante trouxe alguns questionamentos sobre a migração de Mimas. Neste trabalho, investigamos a migração de Mimas considerando a reologia de Maxwell para a maré em Saturno e o modelo de fluência para a maré em Mimas. Descrevemos Saturno como um corpo de duas camadas e analisamos dois casos possíveis: quando há apenas dissipação no núcleo sólido e quando há apenas dissipação no envelope de gás. Analisamos a migração de Mimas através da solução numérica das equações médias em um cenário onde o fator de dissipação Q não é constante, a fim de ampliar nossa compreensão sobre sua evolução orbital em cerca de 10^9 anos.

Keywords. Celestial mechanics – Methods: analytical – Planets and satellites: dynamical evolution

1. Introduction

The variations of Mimas's semi-major axis and eccentricity are the results of tidal effects on both the planet and satellite. As we are working with a gas giant and an icy satellite, the modeling for tidal evolution will be different for each body. It is worth mentioning that for this analysis we consider an isolated system, that is, we discard any disturbance due to other satellites and Saturn's rings. The models used to calculate the variations of the two mentioned orbital parameters are described below.

2. Methodology

2.1. Tidal dissipation in Mimas

To describe the tidal dissipation on the satellite we used the creep theory, which is similar to Maxwell's model in rheology due to viscosity being the fundamental parameter in both (Ferraz-Mello 2013). It is worth mentioning that we model Mimas as a homogeneous satellite that is in synchronous rotation with Saturn. Considering Folonier, Ferraz-Mello, & Andrade-Ines (2018) we have

$$\gamma = \frac{3gM_m}{8\pi R_s^2 \eta_m}, \qquad \varepsilon_\rho = \frac{15}{4}\frac{M_m}{M_s}\left(\frac{R_s}{a}\right)^3, \tag{1}$$

and

$$\dot{a}_{\rm Mimas} = -\frac{42R_s^2\bar{\epsilon}_\rho e^2}{5a}\frac{n^2\gamma}{n^2+\gamma^2}, \tag{2}$$

$$\dot{e}_{\rm Mimas} = -\frac{21R_s^2\bar{\epsilon}_\rho e\left(1-e^2\right)}{5a^2}\frac{n^2\gamma}{n^2+\gamma^2}, \tag{3}$$

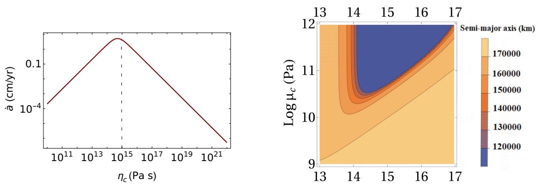
where M_m is the Mimas mass, M_s and R_s are the mass and radius of Saturn, e, a and n are the eccentricity, semi-major axis and mean angular velocity of the satellite, respectively. The parameter γ is the relaxation factor and it is inversely proportional to the viscosity of the satellite. For Mimas we assume $\gamma = 10^{-11}$ s^{-1} (see table 1 of Ferraz-Mello (2013)).

2.2. Tidal dissipation on Saturn

We consider Saturn a two-layer body, so the planet is formed by a viscoelastic core surrounded by a gaseous envelope. Each layer contributes to tidal dissipation separately. FFerraz-Mello, Rodríguez, & Hussmann (2008), we present the equations (Darwin's model) for the tide on the planet:

$$\dot{a}_{Saturn} = \frac{2a}{3n}\frac{9n^2M_mR_s^5}{2M_sa^5}\frac{k_2}{Q}\left(1+\frac{51}{4}e^2\right). \tag{4}$$

$$\dot{e}_{\rm Saturn} = \frac{57neM_mR_s^5}{8M_sa^5}\frac{k_2}{Q}, \tag{5}$$



a)

b)

FIGURE 1. a) The graph shows $\dot{a}$ in cm/year for the viscosity of Saturn's core. The maximum value of $\dot{a}$ is assigned to $\eta_c \approx 10^{15}$ Pa s, denoted by the dotted line. **b)** Variation of μ_c by η_c. The color bar represents the semi-major axis of Mimas in $t = 10^9$ years.

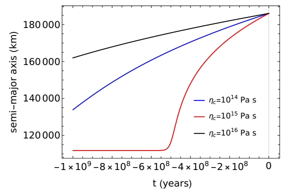


FIGURE 2. Mimas semi-major axis variation in time for the three chosen viscosity values and fixing the other parameters.

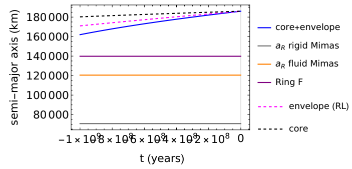


FIGURE 3. Variation of the Mimas semi-major axis over time using the two models (core+envelope) and their contributions separately. We use $\eta_c = 10^{14}$ Pa s.

The ratio k_2/Q is given by the second-order Love number k_2 and the dissipation function Q. In the following subsections, we describe tidal dissipation in the gas envelope and core.

2.2.1. Dissipation in the gas envelope

We analyzed the tidal dissipation in the planet's envelope due to the mechanism known as resonance locking (RL) proposed by Witte & Savonije (1999).

Planets like Jupiter and Saturn present a convective envelope, in which the oscillating fluid generates inertial waves. These inertial waves are considered attractors for tidal energy dissipation (Fuller, Luan, & Quataert 2016). As the planet's internal structure evolves a satellite can get stuck near resonance and migrate. This is the RL mechanism. To describe this mechanism we use the work of Lainey et al. (2020) where the dissipative ratio is given by

$$\frac{k_2}{Q[t]} = \frac{B}{3} \frac{M_s^{1/2} a_0^{1/B}}{G^{1/2} M_m R_s^5 t_0} a[t]^{13/2+1/B}, \tag{6}$$

The parameters a_0 and t_0 are the satellite's current semi-major axis and the planet's age, while B is a constant associated with the motion of the gas in the planet's envelope.

2.2.2. Dissipation in the core

Following Shoji & Hussmann (2017) we use Maxwell's rheology in the study of core dissipation. The following equations are considered

$$Im[k_{2s}] = \frac{k_2}{Q}, \qquad \widetilde{k_{2s}} = \frac{3}{2} \frac{\widetilde{\epsilon} + \frac{2}{3}\beta}{\alpha\, \widetilde{\epsilon} - \beta}. \tag{7}$$

where the constants α and β are depend on the physical parameters of the planet.

3. Results

From the ratios between equations 2 and 4 and equations 3 and 5, we conclude that $\dot{a}_{Saturn} \gg \dot{a}_{Mimas}$ and $\dot{e}_{Saturn} \ll \dot{e}_{Mimas}$. In this analysis we assume $\eta_c = 10^{14}$ Pa s, $\mu_c = 10^{11}$ Pa, $\gamma = 10^{-11}$ s^{-1} and $B = 1/3$ (Shoji & Hussmann 2017; Lainey et al. 2020)

Image **a)** of Figure 1 shows $\dot{a}$ varying with η_c. Note that initially, $\dot{a}$ is directly proportional to η_c and later inversely proportional to η_c, thus forming a peak. We consider the maximum value of $\dot{a}$ when $\eta_c \approx 10^{15}$ Pa s. Regarding the analysis of the parameter space, we built a map varying the two parameters with the greatest influence on the orbital evolution of Mimas, as shown in image **b)**. From the behavior of the graph, we noticed that η_c has greater relevance in the variation of the semi-major axis of the satellite.

From Figures 2 and 1, we constructed Figure 3 where we show the variation of the semi-major axis for three different values of eta_c. The maximum variation of the semi-major axis occurs for $\eta_c = 10^{15}$ Pa s. For $\eta_c = 10^{16}$ Pa s the variation of the satellite's semi-major axis is smaller than for $\eta_c = 10^{14}$ Pa s. Figure 4 shows the variation of the semi-major axis of Mimas considering only the contribution from the nucleus (black dashed line), only the contribution from the gas envelope (pink dashed line), and considering both contributions simultaneously (blue filled line). Let us note that Mimas exceeds the F ring considering the contributions of both layers simultaneously. $\eta_c = 10^{14}$ Pa s was considered. For this value of η_c, the dissipation due to the core has greater relevance in the variation of the satellite's semi-major axis than the dissipation in the gas envelope

4. Conclusions

With the main results we obtained, we can conclude that the orbital expansion of Mimas depends predominantly on the contribution of dissipation in Saturn's core since η_c is the parameter with the greatest power of influence. Depending on the viscosity attributed to the planet's core, this final result is no longer valid. Values below $\eta_c = 10^{14}$ Pa s and greater than $\eta_c = 10^{16}$ Pa s, make dissipation in the gaseous envelope of the planet dominant about the nucleus. As previously mentioned, the maximum dissipation occurs at $\eta_c = 10^{15}$ Pa s, which represents the peak of the curve (see Figure 1).

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