

# Centroiding for truncation-robust wavefront sensor

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**Abstract.** In this new generation of telescopes and giant observatories, adaptive optics systems have become essential for scientific astronomical observation. Unlike space telescopes, ground-based observatories have their imaging of astronomical objects affected by atmospheric turbulence, causing a blurring effect and reducing the resolution of the optical system. These systems require an artificial star for proper operation. However, the image observed by the wavefront sensor in this setup is an elongated point with its luminous intensity dependent on the density profile of sodium. In this case, truncation may exist, which is when the observed object "comes out" the sub-aperture, causing a loss of centering. This error will be present in the future E-ELT telescope, requiring a tool capable of preserving the lost information during truncation.

Resumo. Nesta nova geração de telescópios e observatórios gigantes o sistema de óptica adaptativa tornou-se uma peça essencial para a observação científica astronômica. Observatórios terrestres, diferente de telescópios espaciais, tem a geração de imagens de objetos astronômicos afetada pela turbulência atmosférica, causando um efeito de borrar a imagem reduzindo a resolução do sistema óptico. Estes sistemas requerem uma estrela artificial para seu devido funcionamento, porém a imagem observada pelo sensor de frente de onda neste conjunto é um ponto alongado com sua intensidade luminosa dependente da densidade do perfil de sódio. Neste caso o trucamento pode existir, que é quando o objeto observado "sai para fora" da sub abertura causando a perda do centroiding, erro este que estará presente no futuro telescópio E-ELT, exigindo de uma ferramenta capaz de manter as informações perdidas durante o truncamento.

Keywords. Adaptive Optics – Laser guide – Truncation – E-ELT

## 1. Introduction

In current times, the pursuit of higher resolution in astronomical objects has become a significant challenge, requiring the presence of an observatory or telescope equipped with a grand mirror. For example, we have the Thirty-Meter Telescope (TMT) (Chisholm et al. (2020)), the Giant Magellan Telescope (GMT) (Bouchez et al. (2023)), and the future European Extremely Large Telescope (E-ELT) (Lombini et al. (2012)), which will be the largest among them. However, observations from these ground-based telescopes are limited by atmospheric turbulence, disturbances occurring at various scales in the observatory's optical system, affecting image processing and causing them to appear either in motion or blurred.

Therefore, adaptive optics systems are necessary in virtually all telescopes to achieve image quality close to what would be obtained in space, enabling high-precision astronomical research even in locations with turbulent atmospheres. For the proper functioning of the optical system, the use of a reference star is required, and in certain cases, a laser guide star is employed (Vieira L. - 2018), with sodium layer resonance being a common choice. The observation of this laser in the sky appears as an elongated point due to the vertical depth of the sodium layer, whose atoms are excited by the laser, providing the return light.

In the image formed by atmospheric turbulence, along with the elongated point caused by the artificial star, points near the central region of the mirror are observed in the sub-apertures. Meanwhile, lines are seen in the sub-apertures closer to the mirror. Depending on the degree of turbulence and the noise power, due to the line of sight angle, as depicted in Figure 1, the astronomical object seen in these regions will be in motion, with the probability of "escaping" from the sub-aperture, a phenomenon known as truncation. Truncation is present in giant observatories, precisely because of the size of the primary mirror, and in the future E-ELT observatory, it will be a recurring error. This work proposes a method for determining the centroid (Mello e Pipa (2016)) of observed objects even in the presence of truncation, robust to a high level of noise.

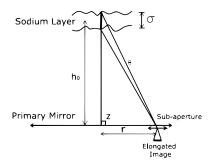
### 2. Adaptive Optics

Considering a flat wavefront without any atmospheric distortion, the observed image is limited only by the diffraction limit of light. This limit is proportional to the Rayleigh criterion ( $\theta = \frac{\lambda}{D}$ ), which can distinguish nearby objects in the field of view. Therefore, in this case, larger telescope apertures are required for higher resolution.

When turbulence is taken into account, the Fried parameter  $(r_o)$  should be used instead of the diameter of the primary mirror. This is because, considering a wavefront distorted by turbulence, processing objects in long exposure results in blurred images, while short-duration images result in motion.

For the effective use of adaptive optics, a laser guide star is commonly employed. This is because, for the proper functioning of the adaptive optics system, bright natural stars near the objects of interest are needed. However, in cases where these natural stars are not available, artificial laser guide stars can be used. The laser interacts with the sodium layer of the Earth's atmosphere (at around 90 to 100 km altitude), causing the region of the layer to become excited and elongated. The laser, when excited, becomes a convenient elongated point for measuring atmospheric distortions, significantly improving optical correction.

In this way, the telescope is limited only by atmospheric turbulence.



**FIGURE 1.** Interaction scheme of the laser guide star with the Sodium layer of the Earth's atmosphere in relation to the primary mirror of the telescope.

The processing of data generated by the wavefront sensor in the presence of atmospheric turbulence is not trivial. In analyzing the sensor-generated points corresponding to an astronomical object, it is observed that points are present in more central regions, while elongated points ("lines") are observed in regions near the sensor's extremes, depending on the density of the sodium profile. Due to the level of turbulence, these points are in motion, causing the elongated spot of the object to "move out" of the sub-aperture, leading to the loss of information in star centroiding.

Centroiding involves detecting the actual center of the extended laser image. Centroiding is crucial to ensure that optical corrections are applied with maximum effectiveness, resulting in sharp and high-resolution astronomical images.

# 3. Applying turbulence to the artificial star

As the basis for the simulation, it was considered that an image of an astronomical object is formed by a Poisson noise distribution operator, given that photon noise follows a Poisson distribution. Additionally, the image is influenced by an operator responsible for the centroiding position at points (x, y), and the sodium layer is assumed to be known, considering the formulation provided in Alexandre's article (Mello e Pipa (2016)).

$$g = PH(\delta_x, \delta_y)f$$

From the following formulation, we can consider that the formed image (g), or in this case, the laser guide star as a reference, is created by a probabilistic distribution of a photon interacting in each pixel, simulating noise and consequently causing truncation. The formulation was applied in the simulation, imposing truncation in the presence of photon noise, with the purpose of estimating the centroiding points in situations of high turbulence experienced by the wavefront sensor.

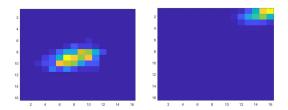
For scanning the center in the elongated point, search tools such as Grid Search and the Newton method were employed. To perform the centroiding search, the probability distribution p is taken into account:

$$p(g|f;x,y) = \prod_{n}^{i=1} \frac{[H(x,y)f]_{i}^{[g]_{i}}}{[g]_{i}!}.e^{-[H(x,y)f]_{i}}$$

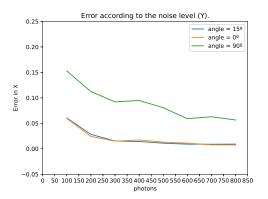
However, for computational implementation, the minimum argument of p is considered precisely because the available functions find the minimum and maximum:

$$\hat{x}, \hat{y} = argmin_{x,y}(-log p(g|f; x, y))$$

Initially, the Grid Search system was implemented, which conducts an exhaustive search for each pixel of the sub-aperture.



**FIGURE 2.** Sub-aperture spot with an elongated point at a 15-degree angle (left) and elongated point experiencing truncation (right).



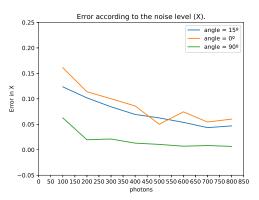
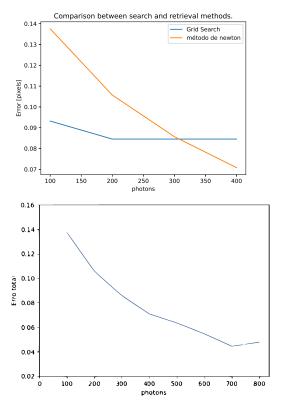


FIGURE 3. Comparison of the error for each point in relation to the photon quantity.

However, this approach was discarded due to the high search time for a specific set of data. Using the Newton method, the centroiding result was obtained in a much shorter period compared to the Grid Search. This is because, starting from an initial point, the Newton method quickly converges to the root when conditions are favorable.

Considering a sub-aperture with 16x16 pixels and a high noise level (Figure 2), three positions of elongated spots at 0, 15, and 90 degrees inclination were analyzed, as shown in Figure 3. Observing first the 'y' axis, a maximum error of 0.15 was obtained, which is extremely robust to noise. However, in the error along the 'x' axis, it is possible to observe a maximum value of approximately 0.161 due to more points focused on their respective axes.

Analyzing the results, emphasizing the 15-degree angle, provides a baseline average across a series of simulations reproducing truncation error, as depicted in Figure 3, with an increase in the number of photons received by the sensor in every hundred simulations.



**FIGURE 4.** In the presence of truncation, the average error in pixels between the actual point and the point found by the search and scan methods.

Considering a scenario without atmospheric turbulence, the average errors due to noise using the Newton method for centroiding search at a 15-degree angle are 0.015.

Considering a noise level of 300 photons, it is observed that the difference between the actual and observed centroiding points is around 0.089. This is a minimal difference for a high level of noise, as compared to other methods such as Grid Search, which has an average error of 0.503 for the same number of photons, and the center of gravity method, with an error of 1.928, as shown in Figure 4.

Therefore, as noise decreases, the error between the results of search tools and the real center decreases. Even at low noise (100 photons), the average error remains around 0.14 pixels.

#### 4. Conclusion

A large telescope aids in capturing more optical energy and achieving better angular resolution (which increases with aperture diameter). However, there is an angular limitation related to light diffraction, and it can worsen when considering seeing conditions. The E-ELT telescope will be equipped with an adaptive optics system due to its considerably large primary mirror, approximately 40 meters in diameter. The larger the diameter of the primary mirror, the more susceptible it is to aberrations and turbulence, both directly proportional to the mirror's size.

The executed simulations demonstrated efficiency in estimating the centroid of the artificial star, showing discrepancies on the order of 0.01. Observing the tools that were used, it was determined that the method that performed best was the Newton method, as the processing time when truncation occurred was shorter than Grid Search. This is crucial in adaptive optics systems, where analyses must be conducted in milliseconds.

Analyzing the results obtained from simulations of how the artificial star will be observed in the new telescope, extremely robust results were obtained even under high turbulence conditions. When applied in an observatory of this magnitude, these results will lead to a very precise analysis in the wavefront sensor, enabling more efficient processing of astronomical images overall.

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