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Numerical investigation of an earth-grazing fireball's close approach

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Abstract. On 1990/10/13 an earth-grazing fireball crossed the Earth's atmosphere, the EN131090 had absolute average magnitude of -6 and lasted 10 secs, with velocity of 41.7 km/sec. It was imaged by two Czech stations of the European Fireball Network. The modified orbit of the remaining material was calculated using the special method of the authors Borovicka and Ceplecha (1992). During the close approaches to the Earth, the meteoroid's orbits is perturbed by the planet's gravity, using Rebound Python package we implemented calculations for that grazing type close encounters back and forth in time, then, the same steps were done running the equations of motion of perturbed two-body problem under a 4th order symplectic integrator with 9 stages.

Resumo. Em 13/10/1990 o fireball rasante EN131090 atravessou a atmosfera da Terra, com magnitude média absoluta de -6 e duração de 10 segundos e velocidade inicial de 41,7 km/s. Ele foi registrado em imagens por duas estações tchecas da European Fireball Network. A órbita modificada do material remanescente foi calculada utilizando o método especial dos autores Borovicka e Ceplecha (1992). Durante o encontro próximo à Terra, sua órbita é perturbada pela gravidade do planeta. Utilizando o pacote python Rebound, realizamos cálculos para esses encontros próximos do tipo rasante ao longo do tempo, coma a aplicação das equações de movimento do problema de dois corpos perturbado com um integrador simplético de 4ª ordem com 9 estágios.

Keywords. Celestial mechanics – Meteorites, meteors, meteoroids – Methods: numerical

1. Introduction

On 1990 October 13th an earth-grazing fireball crossed the Earth's atmosphere, it was observed above Czechoslovakia and Poland and by three independent observers in Czechoslovakia, a radio reflection obtained in Denmark and a photographic report done by two European Fireball Network stations.

Thereso, the remaining could return to space at a different orbit after that short encounter, it was calculated using the special method for long trajectory determination of the authors Borovicka and Ceplecha (1992). The grazing fireball EN131090 had absolute magnitude of -6 and lasted 10 secs, with initial velocity of 41.7 km/sec.

Using Rebound Python package we implemented calculations for that grazing type close encounters back and forth in time, before initial conditions (IC) used for the retrograde integration and the after IC for prograde integration, then, the same steps were done running the equations of motion of perturbed two-body problem under a 4th order Sympletic Integrator with 9 stages.

2. Earth grazing meteors

Grazing meteors are meteoroid debris from comets or asteroids that enter the planetary atmosphere with a near-horizontal path, at a low-angle and perigee very high to the ground, Fig. 1, having only part of their material being ablated during air interaction so the remaining might return to space at a different orbit after that brief close encounter (De Cicco and Szücs-Csillik 2022). Many grazing fireballs have been registered, thanks to growing amateur citizen science projects dedicated to meteors video registering (Bonney et al. 2014; De Cicco et al. 2018).

In general, the Earth-grazings have a luminous path of hundreds to thousands of kilometres moving tangentially through the atmosphere, however, their analyses with the standard assumptions of meteor trajectory (≈ 100 *km*) could bring unrealistic dynamic effects. So, a special method was proposed by Borovicka and Ceplecha (1992) that can be applied to long-grazing meteors such as the 13 October 1990 fireball that after an observed trajectory (> 400 *km*) it returned to interplanetary space.

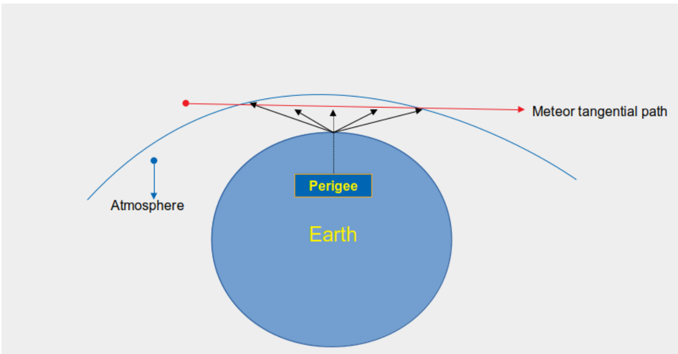

Figure 1. Diagram demonstrating a meteoroid grazing tangentially in the Earth's atmosphere and returns to space. Not in scale.

3. Close encounter

Two dynamic aspects have to be considered about close encounters: the sphere of influence (SOI) and Minimum Orbital Intersection Distance (MOID), as according to mutual distances temporal gravitational forces of Earth can surpass the Sun's attraction (Solar System Central Body) and the distance between the body and the planet orbit become so close that could indicate a future collision. Those encounters have the capacity to change orbits in a way that possible subsequent encounters could not be independent of the occurrence of the previous one (resonant return) (Opik 1976).

It is well known that appropriate time reparametrizations of the equations of motion can be advantageous for the numerical integration of N-body problem. Particularly, this is in the presence of close encounters. In that case, accurate numerical integration requires either decreasing the time-step when approaching a close encounter or applying a suitable time reparametrization (time regularization) that allows one to use constant step size without degrading the accuracy of the computed trajectories. A new fictitious time s and the original time t are usually related through a differential equation $dt/ds = g(u)$, where u is the vector of state variables.

Furthermore, the special perturbation methods became an essential tool in Space Dynamics which exhibits a great advantage: it is applicable to any orbit involving any number of bodies and all sorts of astronomical problems, one such case is, for example, that of Near-Earth Objects (NEOs) dynamics.

Study case: Grazing Fireball EN131090

The all-sky photo of the The bolide EN131090 luminous track during flight is presented in Fig. 2. It had an estimated initial mass of 44 kg, losing only 0.35 kg, with a maximum brightness of -6.45 absolute mag and an almost constant velocity of 41.74± 0.24 km/sec, during all luminous paths it travelled about 409 km.

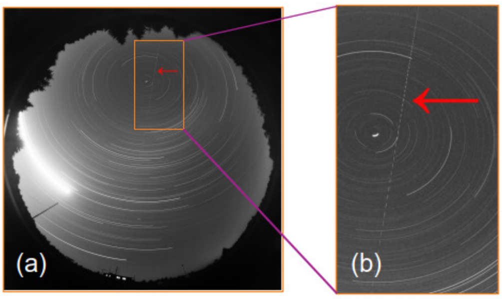

Figure 2. Images: (a) An All-sky picture of the grazing bolide EN131090 captured by the station at Červená hora, situated at Czechoslovakia. (b) The zoomed image shows the interrupted tracklets by a shutter, pointed by a pos-inserted red arrow. Credits: European Fireball Network - RNDr. Pavel Spurný, CSc, Astronomický ústav AVČR.

4. Methods

To investigate the astrodynamic characteristics of a grazer, we need to implement a numerical integration of backward and forward time that can predict the orbital parameters before/after the closest approach, thus, it is suitable to test symplectic integrators, applying before initial conditions (IC) for the retrograde integration and the after IC for prograde integration, analysing possible close encounters and collision studies.

4.1. Neris's 4^{th} order sympletic integrator

So, a non-separable fourth-order symplectic integrator (Neri 1987; Csillik 2004; Szücs-Csillik 2010) to simulate the long-term gravitational dynamics of a meteoroid was tested and analyzed. As the fourth-order symplectic integrator scheme is time reversible because it is symmetric, it ensures the conservation of energy and area-preserving.

Symmetric splitting coefficients with 9 stages are obtained by higher order decomposition of the simple harmonic oscillator. Optimized fourth order symplectic schemes can be useful for achieving high numerical stability allowing large time steps with minimal integration error. High stability of symplectic scheme is particularly important for close encounters in the gravitational N-body system, where inevitably cause high errors can appear.

4.2. Numerical Integration: Before and After Encounter

The semi-major axis is used to compute the total energy of the orbit and together with eccentricity allows us to compute the angular momentum. The meteoroid's angular position in the orbit at a given epoch is defined by the true anomaly. For that reason, the semi-major axis, the eccentricity, and the true anomaly give information about the size and shape of the meteoroid's orbit, and its location in the orbital plane (Roy 1988; Vallado 2013; Anghel et al. 2021). This close approach of the EN131090 meteoroid to the Earth changed its orbital elements in such a way that possible future encounters could not be independent of the occurrence of the previous one, given the concept of resonant returns.

As shown in Fig.3, the meteoroid orbit's inclination angle is almost perpendicular to the ecliptic. We marked its trajectory before the close approach to the Earth with a blue colour, which shows it dancing around the Earth in an elliptical orbit. The trajectory marked in red is the orbit after a close encounter with Earth. It is noticeable how much the proximity of the Earth affected the EN131090 meteoroid's orbit as if it was thrown out.

The simulations showed that among the planets mentioned above, the object had a close approach only with the Earth. Backward integration found only an encounter 706.97 years back, at 0.277549 AU distance and the forward integration got a minimum approach of 0.060304 AU from 74.007 years after the grazing event.

4.3. Regularization Method

It is better to use the KS-regularization (Szebehely 1967) to study grazing meteor orbits, in order to avoid the singularity, we introduce coordinate transformation to blow up the motion around the singularity and time transformation to slow down the motion. A new linear and regular second-order differential equations of motion can used to obtain precise solutions.

As the classical Newtonian equations of motion in rectangular coordinates become singular when the distance between the two bodies, regarded as point masses, tends to zero (close encounter), numerical integrations provide, in the case of close encounters, the singularities can be eliminated by the proper choice of the independent variable.

The basic idea of the regularization procedure (Csillik 2003) is to compensate for the infinite increase of the velocity at collision. A new independent variable, fictitious time, is adopted. The corresponding equations of motion are regularized by two transformations: **time transformation** - as we use a new fictitious time to slow the motion near the singularities and **coordinate transformation**.

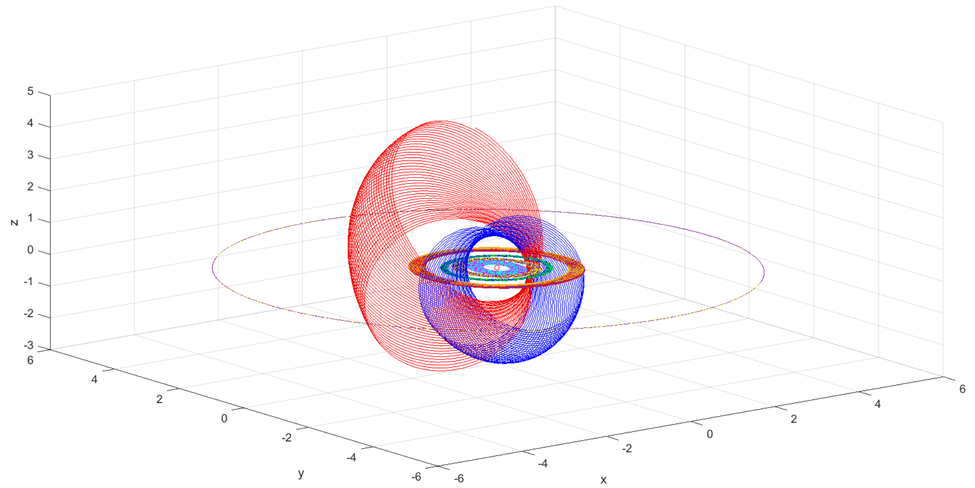

Figure 3. The EN131090 meteoroid orbit - before and after - using Neri 4th order integrator, and its relative position to the orbits of Mercury, Venus, Earth, Mars and Jupiter.

4.4. SRC2BP - Spatial Regularized Circular Two-Body Problem

For elimination of singularities as $\frac{1}{r}$ in the spatial two-body problem, time is considered an independent variable, furthermore, doing the time transformation, $\Delta s = \frac{\Delta t}{r}$, where s is the new variable, so-called fictitious time.

In a fixed reference frame, in space, with relative Cartesian coordinates $q = (q1, q2, q3)$, the two-body problem is described by the Hamiltonian as:

$$H(q_i, p_i) = \frac{\sum_{i=1}^{3} p_i^2}{2} - \frac{\mu}{\sqrt{\sum_{i=1}^{3} q_i^2}}, \tag{1}$$

with the corresponding canonical equations:

$$\begin{aligned} \dot{q}_i &= p_i, \\ \dot{p}_i &= -\frac{\mu q_i}{(q_1^2 + q_2^2 + q_3^2)^{3/2}} = -\frac{\mu q_i}{r^3}, \end{aligned} \tag{2}$$

where q_i and p_i, $i = \overline{1, 3}$ are the canonical coordinates in physical space.

To achieve the regularization in space, which transforms the coordinates and momenta from 3D physical space into the new coordinates and momenta from 4D parametric space (Kustaanheimo-Stiefel (KS) regularization), we adopt the new fictitious time s as a time transformation. We introduce the $L(\mathbf{Q})$ - KS-matrix, which is orthogonal $L^T(\mathbf{Q})L(\mathbf{Q}) = rE$, where E is the identity matrix, and L^T is the transpose of L matrix, $\mathbf{Q}$ is the new coordinates in parametric space.

We introduce the KS-transformation, which transforms the $(q_1, q_2, q_3, p_1, p_2, p_3)$ coordinates and momenta in 3-dimensional physical space into the $(Q_1, Q_2, Q_3, Q_4, P_1, P_2, P_3, P_4)$ new coordinates and momenta in 4-dimensional parametric space.Next step, we adopt the new fictitious time s as a time transformation as $\frac{dt}{ds} = r$.

5. Conclusions

We have developed a numerical method for the 4^{th} order symplectic integrator with 9 stages, which is suitable for close encounter - astrodynamic analysis, for example, in the backward and forward trajectory studies of the grazing bolide EN131090. Moreover, we introduce the KS-regularization method in the close approach analysis. Using the new, regular equations of motion the integrator became faster. The case of the Earth-grazing fireball EN131090 underlines that possible future encounters cannot be independent of the occurrence of previous encounters, and it is therefore important to examine resonance returns.

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