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The reverse variation of the solar day

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Abstract. According to Kepler's first law, the planets orbit the sun in an elliptical path. This ellipse causes the world to slow down as it moves from the point closest to the sun to the point farthest from the sun, and to speed up when the opposite is true. This variation in the planet's speed, combined with the inclination of its imaginary axis, produces the analemma diagram, which can be found by superimposing the positions of the Sun at a given location always in the same unit of time on a clock. The analemma, in turn, describes variations in the duration of the solar day. On some days, these variations in solar days occur in accordance with changes in the speed of the planets, but at other times they get along perfectly. Conversely, some parts of the solar day year show that their periods gradually decrease as the planet slows down, and also increase in periods as the planet speeds up. The purpose of this paper is to demonstrate the occurrence of these reverse variations of the solar day.

Resumo. De acordo com a primeira lei de Kepler, os planetas orbitam o Sol em trajetória elíptica. Essa elipse causa uma desaceleração no planeta quando vai do ponto mais próximo do sol até o ponto mais distante e também causa aceleração quando ocorre o oposto. Essa variação da velocidade do planeta combinada com a inclinação de seu eixo imaginário cria o gráfico do analema, que pode ser encontrado com a sobreposição das posições do sol em um determinado local sempre no mesmo horário de um relógio. O analema, por sua vez, descreve variações nas durações do dia solar. Em algumas datas, essas variações nos dias solares ocorrem de acordo com a mudança de velocidade do planeta, mas em outros momentos, elas se dão de forma perfeitamente invertida e demonstrando períodos menores enquanto o planeta desacelera e também períodos maiores à medida que o planeta acelera. O objetivo deste trabalho é demonstrar a ocorrência dessas variações invertidas dos dias solares.

Keywords. Kepler's laws – Analemma – Equation of time – Solar day

1. Introduction

If we observe the apparent movement of the sun and photograph its position, always at the same time indicated by a clock, at intervals of about ten or twenty days within a year, and we superimpose these photographs, we do not find a straight curve, but a shape resembling the symbol of infinity or the number "eight", this is the phenomenon known as the analema of the Sun.

The reasons for the occurrence of this phenomenon can be explained, at least in part, by Kepler's first law and the fact that the imaginary axis of the planet has an inclination of 23.43917°. If the planet had a perfect circular orbit, there would be no change in velocity, so the only analemma would be a vertical line in the north-south direction. If, in addition to the hypothesis of a perfect circular orbit, there were no inclination of the imaginary axis, then the analemma would be only a single point superimposed on the images of the Sun over a year, always at the same time of a clock. If there were no axis inclination, but only an elliptical orbit, then the analemma would be just a horizontal line, varying only between east and west. With the combination of the elliptical orbit (Kepler's first law) causing a variation in the speed of the planet and the delays and advances of the Sun with respect to our clocks, and the inclination of the imaginary axis causing the north-south variation of the analemma when we observe the successive positions of the Sun for a year, the phenomenon is properly configured.

2. Analemma – Details

The analemma can also be demonstrated in any place where there is sunlight all year round, simply by observing a shadow with a gnomon or a stick placed on the ground or on a wall on which a shadow is projected. If we unite the points of the shadows that appear throughout the year and always at the same time, on a clock, we obtain the graph in the form of an infinity symbol. This is a very didactic way and requires a great simplicity of the materials used to demonstrate the phenomenon, especially for students.

It is obvious that in the observations of the sky, its position varies depending on the latitude of the observer and the time chosen for the observation. It may have different inclinations with respect to the observer in the east (at sunrise) or in the west (at sunset), it may be slightly more inclined at latitudes closer to the equator than at higher latitudes. For a southern hemisphere observer, the main curve is upward, while for a northern hemisphere observer, the secondary curve is upward.

According to the analemma.com team, a third element must be taken into account to explain the origin of the phenomenon. The fact that the major axis of the planet's ellipse does not coincide with the solstices. Indeed, this element may contribute to the composition and configuration of the figure (intentional redundancy) or the analemma curves, but according to the analemma generator of Mandavgane (2015), from the Aryabhat Foundation, the position of the solstices could vary anyway. the solstices could be located at any point of the ellipse where the analemma would be present anyway. The fact that this variation distorts the drawing only slightly prompts other sources to suppress it and mention only the first two elements shown in Figure 1.

Since the sun is at its zenith throughout the tropical zone and changes from one tropic to the other about every six months, we can determine exactly where the sun' is at its zenith during the course of a year, for example, at noon in Brasília, our capital city.

It is important to remember that in this visualization the calendar is inverted with respect to the analemma as observed in the sky. If we look at the map as if we were standing with our backs to the Sun and orbiting the Earth, we can see that at the same time of the clock, the Sun is moving less far to the West, so we

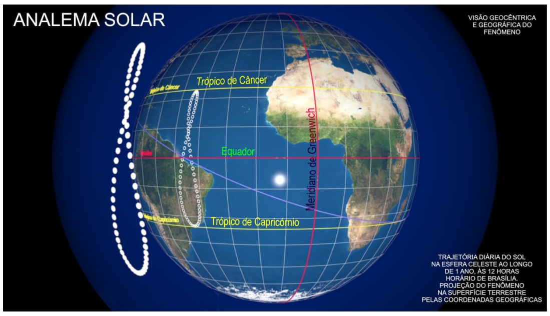

Figure 1. Analemma in its position on the map at noon in Brasília.

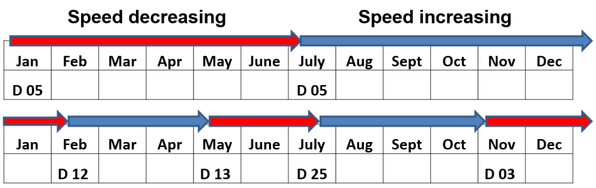

Figure 2. Variation of the speed of the planet Earth and the periods of the solar days in the course of a year.

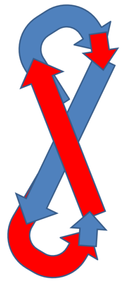

Figure 3. Analemma with indications of variations.

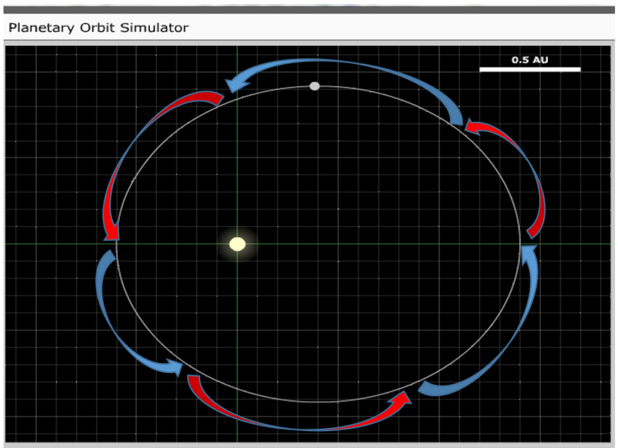

Figure 4. Orbit with approximate dates of variations, inverted and concordant.

can say that it is slower than at another time when it is moving further to the North.

We know that the planet is accelerating as it moves from aphelion to perihelion, and that the opposite is true as it moves from perihelion to aphelion.

Thus, each time the Sun passes the analemma points on its two curves, either north or south, it "slows down" or sets later and later, or rather, in the sense of physics, it shows longer and longer periods because it is not mobile. For analogous reasons, the Sun is accelerated, so to speak, when it passes the analemma points on the two "lines".

Let' us see how a comparison between the times of the known variations of the planetary velocity and the times when the periods of the solar days increase and decrease, in the four inflection points of the analemma, in Figure 2.

The first shows the variations in the speed of the planet in the course of a year, and the second the variations in the periods of the solar days by observing the times of the analemma in which the periods increase and decrease.

The red arrows indicate decreasing speed of the planet and periods when the sun sets later compared to the solar day, or increasing periods. The blue arrows indicate the exact opposite: Acceleration with respect to the planet and periods when the sun sets faster when compared to the solar day. According to the proposed allusion, of course, the sunset earlier each day would be its "acceleration" and the sunset later each day would be its "deceleration".

It is important to note that by "earlier" (or "later") sunset we do not mean the differences caused by the solstices in the north and south, which lead to differences in photoperiod, especially in temperate latitudes. In temperate countries (and even more so at the poles), there is a big difference in the duration of daylight whether we are in winter or summer. What we present here is based on observations made at a latitude of 12.97111° in the south, that is, near the equator and in the tropical zone. This "earlier" or "later" refers to the axis of the planet and not to the occurrence of the solstices, as said.

Usually the literature deals with facts from the temperate zones. The concept of seasons is difficult to apply correctly in the tropics, as shown in The Theory of the Solar Zenith.

To illustrate the occurrence of each inverted variation (and also the times when the variation is normal), we have the following figure of an analemma divided into colored arrows.

The figure was created using arrows from the Word text editor program and therefore does not give the exact shape of the analemma. Its usefulness, however, is that it shows us that the inverted variations in parts occur in exactly the opposite way in the analemma. In other words: In one part of the analemma that has inverted variation, the corresponding part in the other half will have concordant variation. On the "up" (or north) line, the variation is inverted; on the "down" line, the variation is concordant. The lower major curve begins with an inverted variation, so the upper major curve begins with a concordant variation. From aphelion to the end of the upper curve the variation is inverted, from perihelion to the end of the lower curve the variation is concordant. In this diagram, the blue color shows the normal or concordant variation and the red color shows the inverted variation in the Figure 3.

So, with an approximate calendar, we can demonstrate the same events shown in the figure above, including the arrows in the same colors, in the Earth's orbit through the sequence of dates to gain a better understanding of the occurrence of the variations. In Figure 4, the eccentricity of the Earth's orbit is exaggerated for better illustration.

So we can say that the planet changes its speed twice a year, while the solar day changes its periods four times. Three times the variations are reversed and three times they coincide with the speed variations of the planet, i.e. we have six variations during the year.

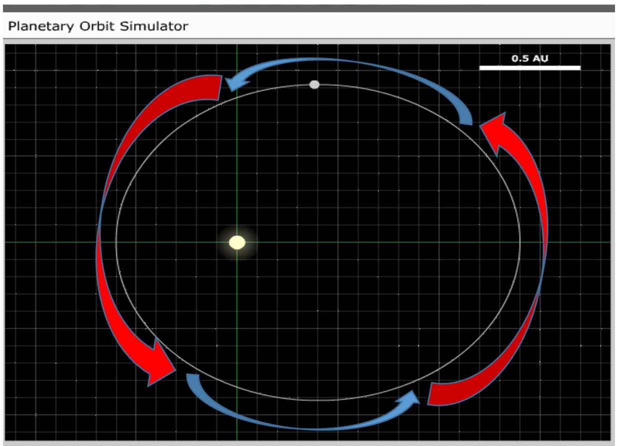

Figure 5. Orbit with approximate times of solar day changes at the analemma inflection points.

The left arrow points to the December solstice, which is summer for the northern hemisphere and winter for the southern hemisphere, which is near perihelion, and the right arrow points to the northern solstice, where these seasons are reversed with respect to the hemispheres of the Earth, which is near aphelion. The angles are different because of Kepler's second law, since the solstices occur about 14 days before the culmination points, aphelion and perihelion, and the planet moves more slowly in its orbit at aphelion than at perihelion. So at perihelion, which is the same 14-day period, the traffic in the orbit is greater.

If we consider only the four variations in the periods of the solar days, without comparing them with the passes through aphelion and perihelion demonstrated by the four inflection points of the analemma, we can also show in the orbit with the approximate calendar in Figure5.

Here the red colors have been used to show the approximate points of the orbit where the analemma is on the two "curves" and the blue color to show when it is on the "lines", so to speak.

3. Pythagoras in Orbit

The Sun is 109.2986 times larger in diameter than the Earth, and we know that at the time of the southern solstice the orbit is at a "highest" point, concentrating the rays at the zenith in the Tropic of Capricorn. In the north the planet goes "below" the Sun, concentrating the rays in the Tropic of Cancer. However, it is necessary to determine a little more precisely the origin of these rays, which fall on the Earth in known geographical coordinates, from the heliographic point of view.

One of the hypotheses would be that the rays falling on the Earth at the northernmost point leave the Suns "equator" and the rays falling on the Earth's south come from a solar "latitude" that is further "north" of it? Could it be that in another hypothesis the shape of the Sun does not interfere with the formation of the analemma, since it is much larger and this practically means that it is a "straight" object which then emits its rays?

To clarify this and many similar hypotheses, we need to bring Pythagoras into orbit. If trigonometry points to 23.43917º declination of the tilt angle on the Earth on the times of the two solstices, and since the distance from the Earth to the Sun on the date of the solstice on June 21, 2021 is ~ 151,715,215.8746 Km, that would be a hypotenuse, the Sun's "Tropic of Cancer" has the declination, also of ~ 23.43917º. The other side of the triangle would have ~ 139,196,116.6722 Km, aligned with the Earths equator until it is "down" or aligned with the exact point

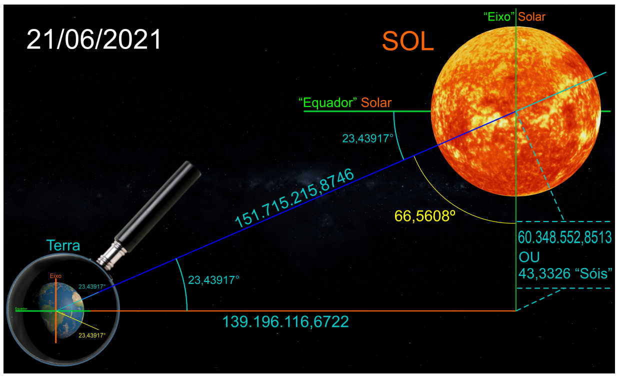

Figure 6. The position of the Earth on June 21, 2021.

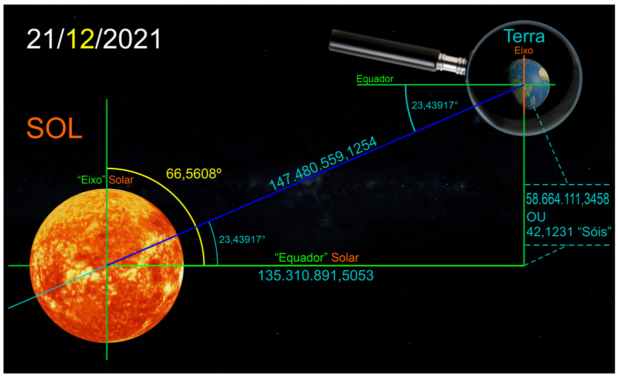

Figure 7. The position of the Earth on December 21, 2021.

of extension of the "axis" to the "south", and the third side of ~ 60,348,552.8513, (from the same point to the Sun's center) to form the 90º angle.

It should be noted that this distance means that the Earth "dips" beyond its equator about 43.3326 times the diameter of the Sun, since its diameter is 1,392,684 km (Figure 6). The proportions between the Earth and the Sun in the next two figures are not observed.

It would be impossible to see the earth if the sun had this size, this is an illustrative figure. Given the distance between the two bodies, it is also impossible to depict the earth and sun in the correct proportions. Also, the size of the Sun is not in proportion to the page that shows how much the Earth "rises" or "falls" in relation to the star.

On the day of the solstice on December 21, 2021, the distance between the Earth and the Sun is ~ 147,480,559.1254 km, another hypotenuse representing the other side of the right triangle measures ~ 135,310,891.5053 km, which would be the side aligned with the Earth's equator and extending exactly to the point of extension of the Sun's "axis" to "north" above, and the third and smaller side with then ~ 58,664,111.3458 km, forming the 90-degree angle in Figure 7.

In this case, this distance shows that the Earth "passes over" the Sun at a "height" of about 42.1231 times its diameter and also marks the "Tropic of Cancer" on the Sun with a declination of ~ 23.43917º.

Of course, such considerations must take into account that the "equator of the Sun" is exactly the point from which the rays strike the Earth at the equinoxes in zenith (Figure 8).

It should also be noted that the "axis" of the Sun has exactly the same orientation as the Earth's axis. Such considerations of Heliometric Coordinates (or Heliographic Coordinates) are similar to those of Geographic Coordinates (and thus of the Earth),

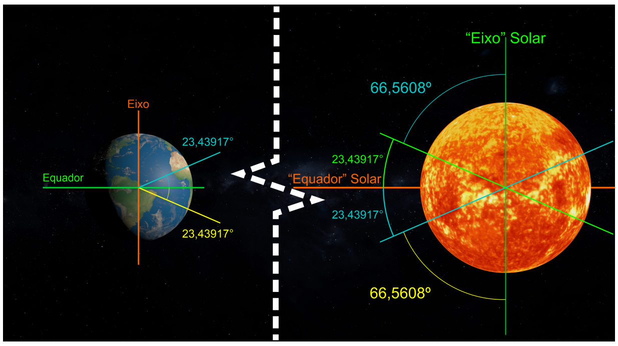

Figure 8. Heliometric coordinates (bodies without the correct size ratio and without showing the correct distance).

they have only the geometric purpose of determining the origin of the rays reaching the planet at the solar zenith at known dates and coordinates.

The imaginary axis of the Sun around which the star revolves is inclined by about 7.25° relative to the axis of the Earth's orbit. The practical effect of this observation is that we see slightly more of the Sun's northern ice cap in September and slightly more of the southern ice cap in March. Thus, it is quite possible that we are considering two coordinate systems for the star: a real one based on observations of the different rotations between its equator and poles, and an imaginary one determined from the projection of the imaginary lines of the planet onto the star for the purposes of geometric data. Similar to the concepts of magnetic north and geographic north of the planet. Similar to the determination of the celestial "equator" for astronomical observation purposes. In Figure 8, the Earth would be in the equinox where its equator projects the identification of the solar "equator".

4. Conclusions and other geometric considerations

The Earth accelerates only in the direction of perihelion or decelerates only in the direction of aphelion in its orbital transit. As it happens, the periods of the Sun's days do not exhibit the same behavior, since the times given for their periods of change (or "velocity") are exactly reversed at certain times of the year, as shown in the preceding pages. There is also no relationship between the slowest or fastest points on the Sun and the slowest or fastest points on the planet, which should be noted when we say that the analemma is formed by variations in the planets orbital velocity.

As shown, comparing the analemma positions with the calendar, we can see that neither the date of maximum variation to the east or west (~ 11/3 and ~ 2/12, respectively) nor the other minor variations to the east or west (~ 5/11 and ~ 7/25, respectively) fall on times when the planet is moving faster or slower. This shows that some geometrical aspects of the Earth-Sun interaction should not be neglected in describing the list of factors contributing to the formation of the analemma.

This work shows that there are variations in the duration of the solar day that vary according to the variation of the planet's velocity in some intervals of the year, but behave inversely in others, which is graphically illustrated by the variations of the analemma itself.

There are inverse variations because the Sun sets later each day between ~ 5/13 and ~ 7/25, the extreme dates of the analemma in the north (and these include the northern solstice and aphelion, ~ 6/21 and ~ 7/5, respectively). If we compare the variation of solar days with the variation of the planets velocity, we see that the planet slows down until ~ 7/5 and at the following moment it starts to accelerate. So there is a coincident variation between ~ 5/13 and ~ 7/5 and an inverse variation from ~ 7/5 to ~ 7/25 because the aphelion date is the time when the planet changes from deceleration to acceleration.

The variations in the planet's rotational velocity are very small, varying in the millisecond range throughout the year. Even if, in a sample of the last five years, we find that each month of January rotates a few milliseconds more than the exact 24 hours, and each month of July a few milliseconds less than the same 24 hours, we can conclude that these variations in the velocity of the Earth' about its own axis do not constitute significant elements in the configuration of the analemma.

It is true that the change of the imaginary radius of the Earths orbit by about 5 million kilometers has some effect on these small variations, but the reasons for them are connected with many other factors, such as moons and tides, and have complex cycles which are not the subject of analysis in this text.

The observation that the speed of the planet about its axis does not vary practically, but the speed of the orbit does, allows us to see some elements that help in the formation of the analemma. If the exposure of the same longitude of the planet in aphelion takes some time, the planet in perihelion is pushed along the orbit by gravity in a more "hurried" way, so that the same longitude facing the Sun has no time to observe the brightness of the star for the same length of time, making the curve in the southern part of the map larger.

The described curve, which starts with the passing of the inflection point ~ 03/11, thus passes through the solstice, the perihelion, and shows an opposite trend only at the other inflection point, ~ 12/02. Thus, if the varying velocity along the orbit is responsible for producing the graph, so is the small or negligible variation in the rotation about the axis itself. If we imagine that the rotation about its own axis is subject to significant variation, this would certainly greatly affect the shape of the graphs and could even cancel out, depending on whether this variation in speed around its own axis is in the same percentages as the variation about the orbit. However, a completely different effect would occur if the change in velocity about the axis happened to be opposite to the change in orbital velocity and by a significant amount. We could even have a concave analemma. So it is necessary to point out that the condition of confrontation between the differences of rotations are also factors causing the analemma.

If we consider the condition of the analemma graph and take into account that the apparent movement of the Sun on the planet is from east to west, the farther east the Sun reaches its zenith at a given hour, the slower the solar day, because the Sun has "wandered a little" on the map. The farther west, the faster, for similar and analogous reasons. The analemma diagram shows us that the Sun is not always slow because the planet is fast, and that the Sun is not always fast because the planet is slow, as one might assume in view of the observed inverse variations.

Another interesting coincidence shown by the analemma is that after aphelion the solar day "slows down" for only a few days (~ 7/5 to ~ 7/25) and then enters the coincident variation described (from ~ 7/25 to ~ 11/3). It also "slows down" for a few more days (~ 1/5 to ~ 2/12, which is also a coincident variation), then "accelerates" (now contrary to the planet's velocity variation) from ~ 2/12 to ~ 5/13. This is because the aphelion and perihelion time, which represent the maximum and minimum velocity points of the planet, do not occur at their inflection points in the analemma diagram.

As we have seen, the variations in analogous parts of the analemma are exactly reversed, one of the lines runs in concordant variation, while the other line runs in reverse variation. One curve starts at the concordant variation, while the other curve starts at the inverted variation.

Perhaps the most important point of geometry that should be emphasized is the spatial variation, i.e., in three dimensions, x, y, and z-axis, of the angle of inclination of the axis with respect to the orbital plane and with respect to the position of the Sun and the empty focal point of the ellipse. Although the Earth is a prograde planet, changes in the incidence of the Sun at zenith along the analemma from south to north or from north to south can produce retrograde effects that help explain, for example, the inverted variations in solar days shown here. The inclination of the axis is unique throughout the shift and the rotation about the axis is also insignificant with respect to the analemma. However, from aphelion to perihelion the Earth begins to receive rays from the Sun in the south, that is, they come from the "north of the Sun" and the opposite happens when the planet goes from perihelion to aphelion. So, on the way there, the spatial-geometric relation of the axes in the orbit is contrary to the spatial-geometric relation on the way back.

The inclination of the axis, which exceeds the orbital inclination of the solstice line of about 12 degrees, marks a lateral plane in horizontal relation to the four analemma inflection points, which are probably related to this horizontal plane and the positions "above" and "below" the orbit on the way to aphelion and on the way back to perihelion. These positions coincide with this plane passing through two points, the projection of the Sun's position in orbit and the projection of the empty focus. On the way back, again passing through these two points, the analemma inflection points are correctly configured.

Another point worth mentioning is that the solar days are "accelerated" on the straight lines and "decelerated" on the analemma curves. The straight lines are at low latitudes while the curves are at higher latitudes. As we have seen, the times of the solstices lie on the curves, just as the high points of the analemma and the times of aphelion and perihelion lie after them. The curves are formed by the passage through the solstices, again the axis plane is responsible. Another consideration is that if the earth were a perfect sphere, the curves would arise at higher latitudes. It is probably possible to calculate the mathematical relationship between the flattening of the planet and these factors, but this will be the subject of further research in the future.

Another interesting and creative assumption from the field of geometry is the idea that the Earth would be a perfectly regular cylinder, with the same diameter as the equator in its circular dimension and the same distance between the poles of the planet in its altitude. If it were to rotate about its imaginary longitudinal axis and describe the same orbital velocities and motions, we would have an analemma entirely different from ours. It would be different in its representation in the sky than in its drawing on the map, and even more, an inhabitant of this "cylinder planet" would see it completely different from us in its sky for the same geometrical reasons.

Finally, not only does the rounded or geoid shape of our planet help create the analemma as we know it, but the geometry of the Sun and its spherical shape also directly affect it. Given the rounded shape of the Earth, which is farther from the Sun at higher latitudes than it is at the equator, and once the motion has taken place and the zenith rays stop touching the line of one of our planet's tropics and approach the equator, given the rounded shape, this contributes to the speed at which these rays reach the planet at zenith through these different latitudes. Of course, if the Earth were a perfect sphere, we would have a different analemma at the higher latitudes of our planet, and an inhabitant of the "spherical planet" would also see a different analemma in its sky.

Since a ray hitting the Earth at its equator will also come from the "equator" of the Sun, the path will naturally be shorter and faster according to the heliometric coordinates, and this also configures the analemma given the temporal variations of the incidence. If the planet is moving. If the ray hits a tropic of the Earth originating in a "tropic" of the Sun, it will certainly be slower in incidence.

Note that the moment when the two bodies approach the equator are exactly the two "lines" of the analemma, which means an "acceleration" of the apparent motion of the Sun. One of them is a moment of deceleration of the planet and, nevertheless, the factor of the rounded geometry of the bodies prevails over the variation of the orbital acceleration of the planet. This refutes the hypothesis that the Sun has a "virtually straight surface" effect given its size and distance.

The heliometric coordinates also provide information about the points of origin of the rays that reach the zenith in the latitudes of the Earth, since we can mark their "tropics" Cancer and Capricorn (with the Sun Capricorn in the north and Cancer in the south, the opposite of the Earth).

The analemma is thus created by a series of factors resulting from the planet's motion around the Sun, in addition to the geometric shapes of the planet itself and the Sun, and not only by the variation of its orbital velocity associated with the inclination of its axis.

A curious point about the heliometric coordinates is that the total displacement of the Earth between one solstice and the next ~ is 119,012,786.0988 km, giving a little over 85 "suns" total displacement. Thus, taking into account that the total displacement takes about six months, the speed of this displacement can be obtained: ~ 27,171.8690 Km/h. Since the maximum orbital velocity is 110,700 km/h, there is a harmonic ratio of 4.0740 between them. Thus, the orbital velocity divided by the velocity in the obliquity between solstices gives a constant. With these data, several other calculations can be made and equations can be established, such as the vectors of this velocity in relation to the orbital velocity, among other findings, such as the same ratio between the hypotenuse of each solstice and the side of the triangle to the solar axis, since they have the same angle, in addition to the trigonometric ratios and the possible projection of the circular shape of the orbit to be confirmed.

It is also possible to determine the "solar tropic" of Salvador with a declination of 12.97111° in the direction of the "north" of the Sun.

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