Boletim da Sociedade Astronômica Brasileira, **34**, no. 1, 253-254

Lorentz invariance violation

When the speed of light depends on the energy of the photon

T. L. Mundo[1] & V. de Souza[1]

[1] Instituto de Física de São Carlos, Universidade de São Paulo, e-mail: tales.mundo@usp.br, vitor@ifsc.usp.br

Abstract. In the 20th century, the union of quantum mechanics with special relativity gave rise to one of the theories with greatest predictive power to date. It is known, however, that this is not the complete description of the Universe, since it does not consider the gravitational interaction. Several attempts to develop a quantum description of gravitaty suggest that the Lorentz invariance (LI), cornerstone of special relativity, is violated (LIV). Such violation would lead to a series of exotic phenomena, such as the dependence of the speed of light on the energy of the photon, a phenomenon addressed in this work.

Resumo. No século XX, a união da mecânica quântica com a relatividade restrita deu origem a uma das teorias físicas com maior poder preditivo já criadas. Sabemos, entretanto, que ela não é uma descrição completa do Universo, já que não considera a interação gravitacional. Diversas tentativas de desenvolver uma descrição quântica da gravitação sugerem que a invariância de Lorentz (LI), pilar central da teoria da relatividade restrita, seja violada (LIV). Tal violação levaria a uma série de fenômenos exóticos, como a dependência da velocidade da luz com a energia do fóton, fenômeno abordado neste trabalho.

Keywords. Gamma rays – Cosmology – Gamma-ray burst

1. Introduction

At the beginning of the 20th century, the foundations of physics were shaken as the newly developed electromagnetism was shown to be incompatible with the established framework of classical mechanics. The problem was that the equations of electromagnetism were not invariant under a Galilean boost, but were under a new transformation, now labelled Lorentz boost. It was later confirmed that the speed of light was the same for all inertial frames, further cementing the need for a new framework. In 1905, Einstein published in his famous paper a new scheme on which dynamics could be studied, the special relativity, a theory that have Lorentz invariance (LI) at its core. Years later, this theory was expanded into general relativity, our best understanding of gravity to date. Special relativity was also incorporated in quantum mechanics throughout the century, giving rise to the most precise theory ever built. These two theories, nonetheless, are incompatible with each other and no satisfactory quantum description of gravity is yet known. Numerous candidates exist, and several suggest that Lorentz invariance is violated (LIV) (Alfaro (2005)), leading to a set of exotic phenomena that could, in principle, be observed, such as the dependency of speed of light on photon energy. Here, we show how introducing a LIV term in the photon dispersion relation leads to variation on the speed of light and discuss how astrophysical gamma-ray photons can help in the search for LIV.

2. Derivation

2.1. Flat, static Universe

The more fundamental aspects of LIV are not our concern in this work. A detailed discussion can be found in the literature (**?**, **?**), and here we focus our attention on some LIV consequences. One way to analyse such consequences is via the dispersion relation of a free particle. The relativistic relation is altered (**?**) as in Eq. 1.

$$E^2 = m^2c^4 + p^2c^2 - \sum_n \xi_n \delta^{(n)} (pc)^{n+2} \tag{1}$$

The $\delta^{(n)}$ coefficients are related to the energy scale where quantum effects of gravity become important and can be taken to be $\delta^{(n)} = 1/(E_{LIV})^n$, whilst ξ_n can take the values ± 1 and its physical meaning shall be evident shortly. We expect E_{LIV} to be close to the Planck energy, making the effects of these corrections minute. This allows us to truncate the series in Eq. 1 to leading-order n. As we are working with photons, we can take $m = 0$, arriving at Eq. 2.

$$E = pc\left[1 - \xi_n \frac{1}{2}\left(\frac{pc}{E_{LIV}}\right)^n\right] \tag{2}$$

Now, we can find the group velocity of the photon by using $v = \partial E/\partial p$, as in Eq. 3. Since the $pc << E_{LIV}$, we can take $E \approx pc$, making the dependence of the speed of light on energy explicit. It's worth noting that if we take $E_{LIV} \to \infty$, we recover $v = c$. The physical significance of ξ_n is now clear: if $\xi_n = +1$, $v < c$ and we have "subluminal" LIV; if $\xi_n = -1$, $v > c$ and we have "superluminal" LIV.

$$v^{(n)} = c\left[1 - \xi_n \frac{n+1}{2}\left(\frac{pc}{E_{LIV}}\right)^n\right] = c\left[1 - \xi_n \frac{n+1}{2}\left(\frac{E}{E_{LIV}}\right)^n\right] \tag{3}$$

Since the speed of light now varies with the photon energy, two photons with distinct energies emitted simultaneously from the same source will travel a distance d in different amounts of time. If we consider two photons with energies E_h and E_l, $E_h > E_l$, the time delay acquired travelling d is described in Eq 4.

$$\Delta t^{(n)} = t_2^{(n)} - t_1^{(n)} = \xi_n \frac{d}{c}\left[\frac{n+1}{2}\ \frac{E_h^n - E_l^n}{E_{LIV}^n}\right] \tag{4}$$

Equation 4 shows the time delay is proportional to the travelled distance and goes with a power of the photons' energies. Thus, astrophysical sources of highly energetic photons are of particular interest in the search for this kind of time delay. For local sources, such as galactic pulsars (Ahnen (2017)), Eq. 4 is perfectly fine, but it fails for distant sources since it does not account for the expansion of the Universe. A more general expression will be deduced next.

2.2. Expanding Universe

In this section, we will derive a new time delay expression assuming the FLRW metric. The discussion for an expanding Universe is quite similar to the flat one, with the replacement $E \to E/a(t)$, with E now being the observed energy of the photon during detection. The $a(t)$ function is the scale factor, a parameter that tracks the expansion of the Universe. It is conventionally set to 1 at the present. Using Eq. 3 with this slight alteration, we can now calculate the comoving distance of a photon, given by Eq. **??**.

$$\chi = c\int_0^t \frac{dt'}{a(t)}\left[1 - \xi_n \frac{n+1}{2}\left(\frac{E}{a(t)E_{LIV}}\right)^n\right] \tag{5}$$

It is more convenient to change the scale factor for the redshift, using the relation described in Eq. **??**. Identifying $\dot{a}/a$ as the Hubble parameter, $H(z)$, we arrive at Eq. **??**

$$1 + z = \frac{1}{a(t)} \to \frac{dz}{dt} = -\frac{\dot{a}}{a}\frac{1}{a} \tag{6}$$

$$\chi = c\int_0^z \frac{dz'}{H(z')}\left[1 - \xi_n \frac{n+1}{2}\left(\frac{E}{E_{LIV}}\right)^n (1+z')^n\right] \tag{7}$$

The expression for the Hubble parameter highly depends on the model in use. In the FLRW metric, it is given by the first Friedmann equation (Eq. **??**).

$$\left(\frac{H(z)}{H_0}\right)^2 = \frac{\Omega_{R,0}}{a^4} + \frac{\Omega_{M,0}}{a^3} + \frac{\Omega_{k,0}}{a^2} + \Omega_\Lambda \tag{8}$$

There are some parameters in Eq.**??**: $\Omega_{R,0}$ is the radiation density parameter in the present; $\Omega_{M,0}$ is the matter (normal and dark) density parameter; $\Omega_{k,0}$ is the curvature density parameter; Ω_Λ is the dark energy density parameter. In the standard cosmological model, the Λ-CDM, observations indicate a flat universe dominated by matter and dark energy, with $\Omega_\Lambda = 0,685$ and $\Omega_{M,0} = 0,315$. Using Eq. **??**, we have $H(z) = H_0\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}$.

Now, consider again two photons with energies E_h and E_l, $E_h > E_l$, emitted simultaneously of the same source. The higher energy photon will have a slight difference Δz in its redshift due to the variation in speed of light. Note that the sign of Δz depends on the sub or superluminal nature of the violation. Since the comoving distance for the two photons are equal, we can match both. Breaking the integration intervals and regrouping, we get Eq. **??**.

$$\int_z^{z+\Delta z} \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}}\left[1 - \xi_n \frac{n+1}{2}\left(\frac{E_h}{E_{LIV}}\right)^n (1+z')^n\right] = \\ = \xi_n \frac{n+1}{2}\left(\frac{E_h^n - E_l^n}{E_{LIV}^n}\right)\int_0^z dz' \frac{(1+z')^n}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}} \tag{9}$$

As the difference in redshift is considered small, we want **??** to be linear in Δz. Expanding around $z = 0$, we have $1/\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda} \approx 1/\sqrt{\Omega_M + \Omega_\Lambda} - 3z\Omega_M/2(\Omega_M + \Omega_\Lambda)^{(3/2)}$, allowing us to take only the first term. Now, using $\Omega_M + \Omega_\Lambda \approx 1$ and $\Delta t = \Delta z/H_0$, we arrive in our expression to the time delay, described in Eq. **??**.

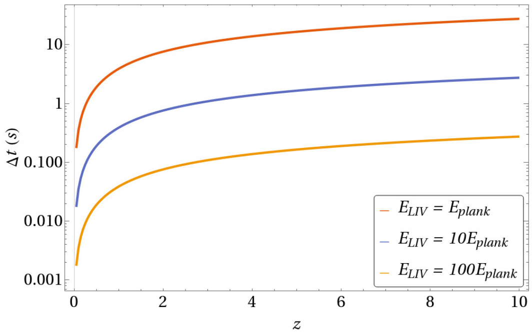

Figure 1. Estimation of time delay for $\Delta E = 100$ GeV and $n = 1$

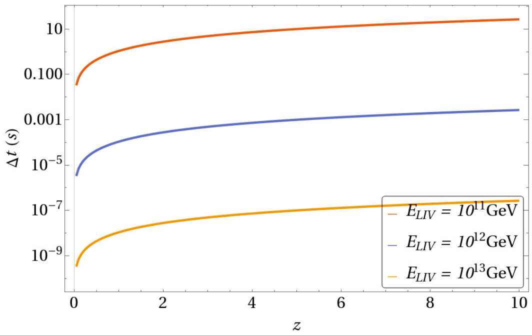

Figure 2. Estimation of time delay for $\Delta E = 100$ GeV and $n = 2$

$$\Delta t^{(n)} = \xi_n \frac{n+1}{2H_0}\left(\frac{E_h^n - E_l^n}{E_{LIV}^n}\right)\int_0^z dz' \frac{(1+z')^n}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}} \tag{10}$$

Typically, E_h varies from 10-100 GeV, while E_l is around hundreds of keV, so $E_h^n - E_l^n \approx \Delta E^n$. Equation **??** allows data from cosmological photons, such as from GRBs, to impose limits in E_{LIV}. One of the best restrictions was found by **?**, with $E_{LIV}^{(1)} > 7.6\, E_{plank}$ for $n = 1$ and $E_{LIV}^{(2)} > 1.3 \times 10^{11}$ GeV for $n = 2$. Figures **??** and **??** show some estimates for the time delay for a typical energy difference. In principle, it's observable.

3. Conclusion

The search for violations of Lorentz invariance showed that, for astrophysical sources of gamma rays, it is possible to observe the time difference between photons of different energies. Equation **??**, however, doesn't account for other possible sources of delay. In the future, we hope to analyse such sources and apply the formalism presented here to real data.

Acknowledgements. This project was funded by FAPESP (2022/02025-0 & 2021/01089-1).

References

Ahnen, M. L. et al., 2017, The Astrophysical Journal Supplement Series, 232, 9

Alfaro, J., 2005, Physical review letters, 94, 22130

Colladay, D. & Kostelecký, V. A., 1997, Physical Review D, 55, 6760

Colladay, D. & Kostelecký, V. A., 1998, Physical Review D, 58, 116002

Mattingly, D., 2005, Living Reviews in relativity, 8, 1

Vasileiou, V. & Jacholkowsk, A. & Piron, F. & Bolmont, J. & Couturier, C. & Granot, J. & Stecker, F. W. & Cohen-Tanugi, J. & Longo, F., 2013, Physical Review D, 87, 122001