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# Numerical study of the correlation between mean motion resonance and planetary mass ratios

V. Alencastro[1] & A. Rodríguez[2]

[1] Observatório Nacional e-mail: viviamguimaraes@on.br
[2] Observatório do Valongo, Universidade Federal do Rio de Janeiro e-mail: adrian@astro.ufrj.br

**Abstract.** In this project, a series of numerical simulations were run for hypothetical systems composed of a star and two planets with initial orbital periods in ratio 2/1. It was studied how planetary mass ratios correlate with the extension of mean motion resonance (MMR) occurrence regions in the space of orbital eccentricities. The results obtained include dynamic maps of symmetric 2/1 MMR, apsidal corotation and apsidal corotation resonance. These maps constitute a catalogue that can be used to estimate the coplanar dynamics of observed systems with orbital periods ratios close to 2/1 and eccentricities below 0.3. As application example, system TOI-216 was numerically integrated and its dynamic patterns were compared with those predicted using the catalogue.

**Resumo.** Neste trabalho, uma série de simulações numéricas foram realizadas para sistemas hipotéticos de uma estrela e dois planetas com períodos orbitais iniciais em razão 2/1. Estudou-se como as razões de massas planetárias se correlacionam com a extensão das regiões de ocorrência de ressonância de movimentos médios (RMM) no espaço de excentricidades orbitais. Os resultados obtidos incluem mapas dinâmicos de regiões de RMM, de corrotação apsidal e de ressonância de corrotação apsidal simétricas. Os mapas constituem um catálogo que pode ser utilizado para estimar a dinâmica de sistemas observados coplanares, com razões de períodos orbitais próximas à relação 2/1 e excentricidades até 0.3. Como exemplo de aplicação, o sistema TOI-216 foi integrado numericamente e sua dinâmica foi comparada com as estimativas fornecidas pelo catálogo.

**Keywords.** Celestial mechanics – Methods: numerical – Planets and satellites: dynamical evolution – Catalogs

## 1. Introduction

Several planetary systems have been discovered in the past years and new observational missions are still under development aiming to find exoplanets in habitable zones. In the search for conditions that are suitable for life, dynamically stable systems are of special interest. In this context, mean motion resonance in multiplanetary systems is an important dynamic phenomenon, since it affects their long term orbital evolution.

A MMR can exist when the orbital periods of planets are proportional to one another, satisfying a simple ratio of integers. Depending on the configuration of the planets in (or near) MMR, a system may manifest greater or lesser variations in its orbital elements over time. Also, the occurrence of MMR can indicate processes of planetary migration that have possibly happened during the formation of a planetary system, which is investigated by Beaugé, Michtchenko & Ferraz-Mello (2006), Deck & Batygin (2015) and others.

Most of the likely resonant systems observed so far have orbital periods ratios close to 2, which is shown in the statistics of Winn & Fabrycky (2015). Considering this fact and also that resonances can only be most globally investigated using numerical tools, our goal is to specify generic resonance occurrence regions for the 2/1 MMR for a representative range of orbital parameters and planetary mass ratios and thereby provide a catalogue of dynamic maps. The latter shall then be used to estimate the dynamics of any near resonant system recently detected and still not completely characterized.

## 2. Methods

The methodology applied here to numerically explore the dynamics of various possible resonant systems consists of defining a vector of planetary mass ratios, $r = m_2/m_1$ (in which $m_2$ and $m_1$ are the masses of the outermost and innermost planets, respectively) and generating triplets of dynamic maps for each ratio value. For a given ratio, the color scales of each map separately indicate the oscillation amplitudes of $\sigma_1$ and of $\Delta\varpi$ (i.e. $\varpi_2 - \varpi_1$), as well as the occurrence of apsidal corotation resonance (ACR). $\sigma_1$ is the critical angle of the 2/1 MMR associated with $\varpi_1$ ($\sigma_1 = 2\lambda_2 - \lambda_1 - \varpi_1$), $\varpi_i$ are longitudes of pericenter and ACR means that both $\sigma_1$ and $\Delta\varpi$ oscillate.

The maps' grids determine initial conditions of eccentricities and orbital angles. The latter are calculated such that the initial values of $\Delta\varpi$ and $\sigma_1$ are either 0° or 180°, for the focus is on symmetric MMRs, and the eccentricities range to 0.3, which is in accordance with observational data for most systems near the 2/1 MMR. Also, initial mean anomalies and inclinations are zero for all simulations, since one aims to investigate the long term evolution of approximately coplanar systems.

For each map triplet, a code calculates the amount of simulations that displays different types of long term dynamics. The percentage of occurrence of each type is then plotted against mass ratio, in order to evince any possible correlation between these two concepts.

This paper presents only one example of map triplets in section 3, but a total of 312 maps were generated, constituting a catalogue that is valid for planetary mass ratios ranging from 0.05 to 20. The numerical integrations were performed using the computation cluster Lobo Carneiro from the Federal University of Rio de Janeiro, as well as the national computer cluster Santos Dummont.

## 3. Results

This section presents a sample of the generated catalogue. Figure 1 features an example of map triplet for a hypothetical system with a sun-like star, two planets with mass ratio 6 and orbital pe-

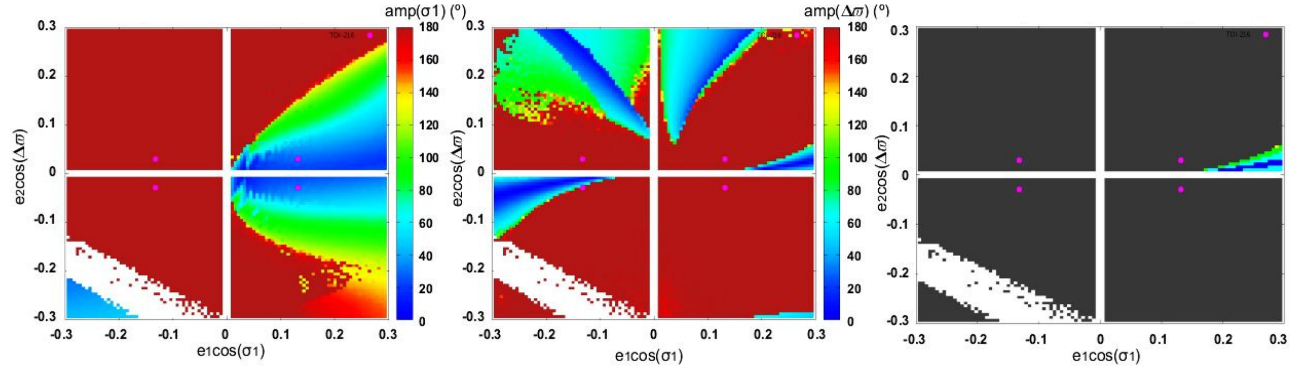


**Figure 1.** TOI-216's eccentricities marked as dots on maps of $\sigma_1$ libration amplitude, $\Delta\varpi$ oscillation amplitude and ACR regions, respectively. Maps were generated for a hypothetical system of mass ratio 6, periods ratio 2 and a sun-like star. Integration time: 4000 years. White represents unstable simulations, red means mapped angle circulations and gray means non-ACR dynamics.

riods ratio 2 (the innermost planet having an initial semi-major axis of 0.05 au). Marked as dots on the triplet are the eccentricities of both planetary orbits of system TOI-216 provided by Kipping et al. (2019). According to this reference, TOI-216 has a planetary mass ratio close to 6 and orbital periods ratio close to 2.

The first map on Figure 1 indicates that TOI-216 is inside resonance regions for the right side quadrants, which means that it may display symmetric resonance if its orbital angles are such that the system is close to having $\sigma_1 = 0°$ and $\Delta\varpi = 0°$ or 180° at a given moment. In turn, the map indicates the system cannot be resonant for an equilibrium point of $\sigma_1 = 180°$. The central map on Figure 1 suggests that TOI-216 may present, though unlikely, oscillation of $\Delta\varpi$ only if its orbital angles are such that they approach the equilibrium points of $\sigma_1$ and $\Delta\varpi$ equal 180°. Finally, the third map indicates that this system is outside the parameters region that allows for symmetric ACR in any angular case.

Simulating TOI-216 individually, with its complete set of parameters proposed by Kipping et al. (2019) and both values of symmetric $\Delta\varpi$, it is evident that the system behaves consistently with the dynamics indicated by the catalogue triplet, which is a first validation of the catalogue as an estimation tool for systems near the 2/1 MMR.

Figure 2 in turn presents an overview of all map triplets of the catalogue and the occurrence of symmetric ACR for each mass ratio tested. To automatically calculate the amount of simulations in ACR, it was necessary to determine arbitrary criteria for oscillation amplitudes. Therefore, there are six different criterion values, in which the blue curve represents librations with very low amplitudes of both angles, $\Delta\varpi$ and $\sigma_1$. One easily concludes that librations with higher amplitudes are more frequent for systems with mass ratios below 2, that very low amplitudes occur for mass ratios close to 1.5 and that mass ratios much greater than 1 do not allow for symmetric ACR. It is important to highlight that these results are only valid regarding orbital eccentricities below 0.3.

## 4. Conclusion

In this project, a series of numerical simulations were run for hypothetical systems of a star and two planets near the 2/1 MMR. It was studied how planetary mass ratios correlate with the symmetric MMR regions in the space of eccentricities. For these below 0.3, we conclude that planetary mass ratios close to 1.5 favor symmetric ACR with very low libration amplitudes, whereas ratios near 20 disfavor symmetric ACRs in general.

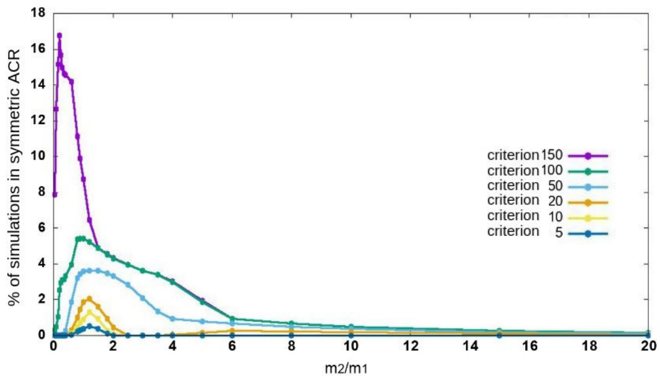


**Figure 2.** Percentage of simulations with $\sigma_1$ and $\Delta\varpi$ amplitudes smaller or equal to criterion values (in degrees) vs. planetary mass ratios. The graph presents the correlation of symmetric ACR occurrence and mass ratios, indicating a higher chance of low amplitude librations for ratios near 1.5.

The project's overall results constitute a catalogue of 312 dynamic maps, which display symmetric MMR, apsidal corotation and ACR regions for a representative range of planetary mass ratios. Observed system TOI-216 was individually simulated and validated the utilization of the catalogue as an estimation tool. Additionally, our complete study shows the generated maps do not depend on individual semi-major axis values or stellar mass, as long as the proximity to the symmetric 2/1 MMR exists. Finally, the catalogue may be utilized to estimate whether resonant dynamics is possible for observed systems near the 2/1 MMR and its full content will soon be publicly available at the Valongo Observatory's digital library of thesis along with a more comprehensive body of results and analysis.

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