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Alfvén waves damping as a heating source for differentially protostellar rotating disks

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Abstract. The damping of Alfvén waves has been proposed as a viable mechanism to increase the ionization fraction in protostellar accretion disks, thus, ensuring that the Magnetorotational Instability is efficient in a larger region of these disks. We have previously investigated the role of multiple mechanisms of damping of Alfvén waves in heating these structures, including the resonant absorption of surface Alfvén waves and the turbulent damping. Taking a step further, we continue this work by considering the effect of differential rotation, characteristic of Keplerian accretion disks, in the heating rate associated with resonant absorption. This dynamics could, in principle, trigger a shear instability that would significantly change the environment of the non-uniform layer where the resonance is expected to develop. As already done by many authors in the literature, to study this process we perform a linear analysis of the disk, by means of a local approach. Additionally, we substitute the true discontinuity by a thin transition layer, where the Alfvén resonance point is assumed to be located, in order to achieve analytical results. We obtain that, indeed, the efficiency of the mechanism is substantially affected by the rotational shear, since besides the resonant absorption, now the effects related to the Keplerian shear are also considered, which could decrease the final rate. This behaviour is most likely explained by the stability obtained, at least in linear levels, of Keplerian flows, when only the rotational shear is accounted for igniting some sort of instability.

Resumo. O amortecimento de ondas Alfvén já foi proposto como um mecanismo viável de aumento da fração de ionização em discos de acreção protoestelares, assegurando, dessa maneira, a eficiência da Instabilidade Magneto-rotacional em uma maior região do disco. Nós já estudamos previamente o papel de diversos mecanismos de amortecimento de ondas Alfvén em aquecer essas estruturas, incluindo a absorção ressonante de ondas Alfvén de superfície e o amortecimento turbulento. Aprofundando um pouco mais nesse tópico, nós continuamos esse estudo agora considerando o efeito da rotação diferencial, característica de discos Keplerianos, na taxa de aquecimento associada com a absorção ressonante. Essa dinâmica poderia, em princípio, originar uma instabilidade devido ao cisalhamento de velocidades que poderia alterar significativamente o ambiente da camada não-uniforme onde espera-se que a ressonância ocorra. Como realizado por diversos autores, nós realizamos uma análise linear do disco, através de uma abordagem local, para investigar esse processo. Adicionalmente, nós substituímos a descontinuidade real por uma fina região de transição, onde o ponto de ressonância Alfvén está localizado, com o objetivo de obter resultados analíticos. Nós obtemos que, de fato, a eficiência dos mecanismos é substancialmente afetada pelo cisalhamento de velocidades devido à rotação, já que além da absorção ressonante, os efeitos relacionados ao cisalhamento Kepleriano também começaram a ser considerados, o que pode diminuir a taxa de amortecimento final. Esse comportamento é muito provavelmente explicado pela estabilidade obtida, pelo menos em uma abordagem linear, de fluidos Keplerianos, quando apenas o cisalhamento de velocidades é considerado no aparecimento de algum tipo de instabilidade.

Keywords. Accretion, accretion disks – Magnetohydrodynamics (MHD) – Stars: pre-main sequence

1. Introduction

The evolution of young solar type stars is deeply affected by its surround disks and, in the current understanding, those structures are responsible for promoting the matter transport towards the central object, through a process intrinsically related to the angular momentum (AM) transport in those disks. Among the various mechanisms that was proposed to explain this AM transport (e.g. gravitational and hydrodynamic instabilities), one of the most accepted is the Magnetorotational instability (MRI, Balbus & Hawley 1991), which requires the disk particles to have a minimum ionization fraction in order to be effective. Gammie (1996) noticed, however, that this requirement may not be met in the whole disk. Indeed, near the midplane regions, where the ionization fractions are low, he found that this instability was unable to act and the flow remained in a laminar state. This region was then named *dead zone* and, since then, has been the subject of multiple works (e.g. Fromang, Terquem & Balbus 2002; Martin et al. 2012). The extent of this quiescent region is tightly related to the column density of the disk and is also very sensitive on the non-ideal MHD effects present on the disk (e.g. Dzyurkevich et al. 2013). Additionally, Bai (2011) have found that, in the inner regions of the disk, MRI alone was not able to describe the accretion rates observed and, thus, some extra mechanism should be considered.

With all this in mind, in the present work, following the works by Vasconcelos, Jatenco-Pereira & Opher (2000) and Jatenco-Pereira (2013), we study the extra heating associated with the damping of Alfvén waves in order to increase the disk temperature (and, consequently, its ionization fraction) and thus, ensure that the MRI is effective in a larger region of the disk. We have considered the standard model proposed by Shakura & Sunyaev (1973), which parametrize the disk viscosity as a function of the free parameter, α. Since accretion disks are radially stratified, we also assume that resonant absorption of surface Alfvén waves may occur in these disks, and that this process can dissipate the wave's energy, increasing the disk temperature. Besides, we have also considered that the waves may be damped by a mechanism that couples both the effects of the resonant absorption and turbulent motions, associated with the development of a shear-driven instability due to the rotational shear characteristic of accretion disks, through a process previously proposed by Goossens, Hollweg & Sakurai (1992).

2. The disk model

We assume that the protostellar disk associated T Tauri star follows the standard α-prescription, given by Shakura & Sunyaev (1973), associated with the layered model proposed by Gammie (1996), and can be approximated as being optically thick, geometrically thin and under a Keplerian rotation. Additionally, we adopted the disk opacity as being described by Bell & Lin (1994) opacity law, and postulated, in accordance with Jatenco-Pereira (2013), that, initially, the disk is heated only by the viscous dissipation. Throughout this work, α was taken as constant.

By using the hypothesis that the disk is optically thick, we may assume that it radiates as a black-body, and write the disk effective temperature as:

$$\sigma T_{\rm eff}^4 = \mathcal{F}_\nu = \frac{3\Omega_k^2 \dot{M}}{8\pi}\Big[1 - \Big(\frac{R_i}{r}\Big)^{1/2}\Big], \tag{1}$$

where $\mathcal{F}_\nu$ is the energy flux associated with the viscous dissipation, and the central temperature becomes defined as:

$$T_c^4 = \frac{3}{4}\tau T_{\rm eff}^4, \tag{2}$$

where the hypothesis that the energy is transported radiatively was applied (Hartmann 2009) and τ denotes the disk optical depth. Finally, we assumed hydrostatic equilibrium in order to obtain the vertical structure of the disk.

Now, in order to take into account some extra heating source acting on the disk (the damping of Alfvén waves in the present work), the effective disk temperature becomes defined as:

$$\sigma T_{\rm eff}^4 = \mathcal{F}_\nu + \mathcal{F}_A, \tag{3}$$

where the term $\mathcal{F}_A$ is the dissipated energy associated with the damping of Alfvén waves, which can be written as:

$$\mathcal{F}_A = \int_{-H}^{H} \frac{\Phi\gamma}{v_A}, \tag{4}$$

where H is the disk scale heigth, $v_A = B/\sqrt{4\pi\rho}$ is the Alfvén velocity, γ is the damping rate associated with the extra heating mechanism and Φ indicates the Alfvén wave flux, given by:

$$\Phi = \rho v_A < \delta v^2 >, \tag{5}$$

where $< \delta v^2 >$ is the velocity variance associated with the field. A somewhat similar disk model and procedure, along with a more detailed description of the hypotheses made above, can be found in Vasconcelos, Jatenco-Pereira & Opher (2000) and Jatenco-Pereira (2013).

3. The extra heating mechanism

In order to couple the effects of both the resonant absorption of surface Alfvén waves (SW) and the disk differential rotation, associated with the Keplerian rotation profile of our model, we applied a local approximation and restricted our analysis to the vicinity of a reference point, r_0, which rotates with an angular frequency, Ω_0, where we may define an internal ($x = r - r_0 < 0$) and external ($x = r - r_0 > 0$) medium. We then construct a new cartesian frame, centered at r_0, where the $\hat{\boldsymbol{x}}$ direction is aligned with the radial direction and $\hat{\boldsymbol{y}}$ is aligned with the azimuthal direction. In this new configuration, the radial shear associated with the disk differential rotation becomes simply written as (e.g. Lesur 2021):

$$v_0 = -\frac{3}{2}\Omega_0 x\hat{\boldsymbol{y}}, \tag{6}$$

and the MHD equations are given by:

$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}.(\rho\boldsymbol{v}) = 0, \tag{7a}$$

$$\frac{\partial}{\partial t}\boldsymbol{v} + \boldsymbol{v}.\boldsymbol{\nabla}\boldsymbol{v} = -\frac{1}{\rho}\boldsymbol{\nabla}P + \frac{(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi\rho} - 2\Omega_0\hat{\boldsymbol{z}}\times\boldsymbol{v} + \Omega_0^2(2qx\hat{\boldsymbol{x}} - z\hat{\boldsymbol{z}}), \tag{7b}$$

$$\frac{\partial}{\partial t}\boldsymbol{B} = \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B}). \tag{7c}$$

We then applied to our system perturbations of the form: $\boldsymbol{B} = (\boldsymbol{B_{1x}}, \boldsymbol{B_{1y}}, \boldsymbol{B_0} + \boldsymbol{B_{1z}})$, $\boldsymbol{v} = (\boldsymbol{v_{1x}}, \boldsymbol{v_0} + \boldsymbol{v_{1y}}, 0)$, $\rho = \rho_0 + \rho_1$, $p = p_0 + p_1$, where the perturbed quantities were written as:

$$g(x, y, z, t) = g(x)\exp[i(-\omega t + k_z z + k_y y)], \tag{8}$$

where ω is the frequency of the perturbation, and k_y and k_z are the 'azimuthal' and 'vertical' wavenumbers, respectively. Now, if we assume that the width of the interface, a, where we restrict our analysis, is really thin, $a/r_0 \ll 1$, we may postulate that $\Omega_0 a/v_A \ll 1$, $k_y a \ll 1$. If we make an additional restriction over the wavenumbers of the perturbation, $k_y \gg k_z$, we may obtain, analytically, the damping rate associated with the coupled mechanism, γ_c, which considers both the effects related to the resonant absorption of SW and the assumed turbulence, associated with the development of a shear-driven instability, due to the radial shear of the disk. To analyse if this coupled mechanism is, indeed, more efficient in heating the disk, as expected, we also define the damping rate of the resonant absorption of SW (γ_{SW}), individually, which is obtained when we neglect the motion effects (i.e. $\Omega_0 = 0$) in the investigation explained above. The damping rates for the coupled and resonant mechanisms have the following dependences, repectively:

$$\gamma_c \propto \frac{\rho_e}{\rho_A}k_y a\frac{\big[2\Omega_0(\rho_A/\rho_{0e}\omega_A - \overline{\omega}) + (\overline{\omega}^2 - \omega_{Ae}^2)\big]^2}{\big[\Omega_0(\rho_i/\rho_e - 1) + \overline{\omega}\big](\overline{\omega}^2 - \omega_{Ae}^2)}, \tag{9}$$

and

$$\gamma_{SW} \propto (\rho_e/\rho_A)k_y a\frac{(\omega^2 - \omega_{Ae}^2)}{\omega}, \tag{10}$$

where ρ and ω_A are the volumetric density and Alfvénic frequency ($\omega_A = k_z v_A$) and the subscript e and A denotes that the evaluation is made on the external medium and on the resonant layer, respectively. Also, $\overline{\omega} = \omega + (3/2)k_y\Omega_0 a$ and ω_A, without any subscript, indicates the Alfvénic frequency at the resonant layer.

4. Partial results

Using the procedure described in Sec. 2, and assuming the following parameters for our fiducial model: $M_* = 0.7\,M_\odot$, $\dot{M} = 10^{-7}\,M_\odot\,{\rm yr}^{-1}$, $\mu = 2.33$, $\alpha = 0.01$ e $R_i = 5\,R_\odot$, where M_* is the stellar mass, $\dot{M}$ is the accretion rate, μ is the mean molecular weigth and R_i is the internal disk radius, we obtain that, when viscous dissipation is the only source of disk heating, the central

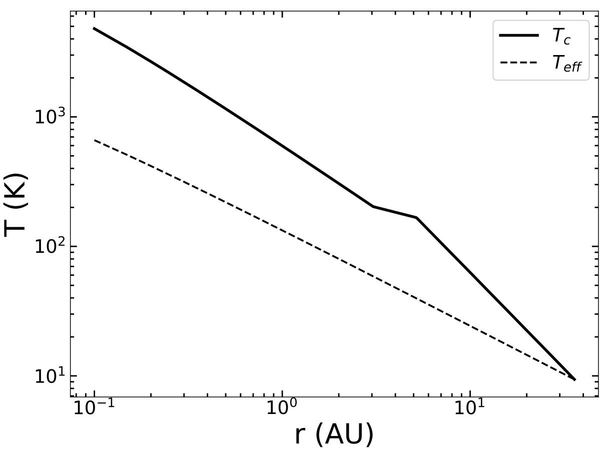

Figure 1. Radial profile of central (thick line) and effective (dashed line) temperatures for the fiducial model, i.e. when only viscous dissipation is responsible for heating the disk.

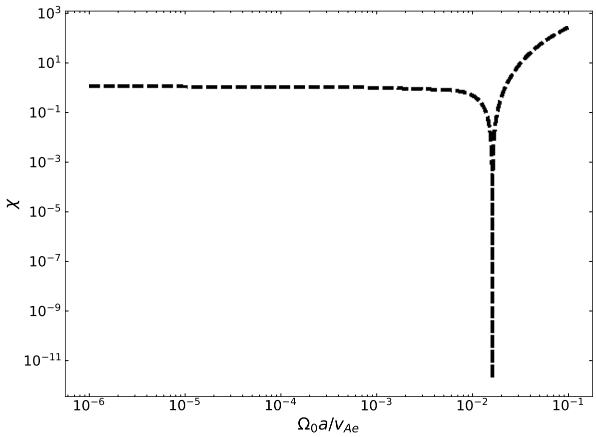

Figure 2. Relative effectiveness of the damping of Alfvén waves mechanisms, $\chi = \gamma_c/\gamma_{SW}$, as a function of the ratio between the radial shear and the Alfvén velocity, $\Omega_0 a/v_{Ae}$.

temperature, near the central object ($r \lesssim 1$ AU), is of the order of 1000 K. The sudden changes in the central temperature behaviour is due to changes in the opacity regime, which is taken to occur abruptly in our model. This behaviour, as well as the radial profile of the central temperature for the fiducial model, is shown in Fig. 1.

Now, before we start investigating how the two extra heating mechanisms impact on the ionization fraction of the disk, we can study the relative effectiveness between them. If we introduce the new dimensionless variable, χ, defined as:

$$\chi \equiv \frac{\gamma_c}{\gamma_{SW}} \propto \frac{\left[2\Omega_0(\rho_A/\rho_{0e}\omega_A - \overline{\omega}) + (\overline{\omega}^2 - \omega_{Ae}^2)\right]^2 \omega}{\left[\Omega_0(\rho_i/\rho_e - 1) + \overline{\omega}\right](\overline{\omega}^2 - \omega_{Ae}^2)(\omega^2 - \omega_{Ae}^2)}, \quad (11)$$

we can find the effectiveness of those mechanisms, as a function of the ratio between the radial shear and the Alfvén velocity in the medium, $\Omega_0 a/v_{Ae}$. This behaviour is shown in Fig. 2: for small values of the radial shear (and, consequently, thinner interfaces, as can be seen in Equation 6), the dominant effect is the resonant absorption of the surface waves, and we find that $\chi \sim 1$ in this regime ($\Omega_0 a/v_{Ae} \lesssim 10^{-2}$). On the other hand, for very high radial shears ($\Omega_0 a/v_{Ae} \sim 10^{-1}$), we do get that the coupled mechanism is more effective, as expected. This is due to the fact that, since we are increasing the shear, the growth rate for the shear-driven instability becomes naturally larger. Now, in order to study how these two limiting regimes impact on the heating of the disk we apply those mechanisms to our disk model, assuming the same initial parameters for the star-disk system of the fiducial model, and the following set of parameters for the heating mechanisms: $k_y a \sim 10^{-1}$, $k_z a \sim 10^{-2}$, $a/r_0 = 10^{-8}$ (when $\chi \sim 1$) and 5×10^{-4} (corresponding to the regime where $\chi > 1$). The correspondent results are shown in Fig. 3.

Figure 3 shows the increase in the central temperatures after the consideration of the two extra heating mechanisms (resonant absorption of SW on the top plots and coupled mechanisms at the bottom) for $a/r_0 = 10^{-8}$ (left pannels) and $a/r_0 = 5 \times 10^{-4}$ (right pannels). It is clear that for $a/r_0 = 5 \times 10^{-4}$ neither mechanisms produces some kind of substancial heating. On the other hand, for $a/r_0 = 10^{-8}$ an increase in the central temperature is observed. This increase, however, is the same for both mechanisms, as expected from the relative effectiveness analysis performed (see Fig. 2). However, a relevant question is why, even in the regime where the coupled mechanism is much more effective than the resonant, a significant heating was not obtained? The answer is quite simple: since we are considering a very thick interface, large a, the effects associated with the resonance becomes negligible, making the resonant absorption of SW much less effective. At the same time, however, the effects associated with the radial shear becomes more and more important, which could then compensate the inefficiency of the resonant damping. But, since our disk model has a Keplerian rotation, the assumed shear-driven instability, which would then be responsible for increase the dissipated energy, does not occur due to the stability of Keplerian flows, at least in linear levels. Thus, the effects associated with the shear-driven instability are also negligible, and we find that the only viable heating source is the resonant damping of SW, for sufficiently thin interfaces. Therefore, if an effective mechanism for disk heating relies on the turbulent dissipation of surface waves energy, the origin of the assumed turbulence is related to other process other than the radial shear associated with the Keplerian rotation of the disk.

5. Main conclusions

In this work, we investigated how the combined action of both the radial shear of protostellar accretion disks and the resonant damping of surface Alfvén waves can impact on the temperature profiles of protostellar disks. We made use of a local approximation and obtained that, while the resonant absorption is indeed effective in heating the disk, and therefore reduce the dead zone extent, for sufficiently thin interfaces, the coupled mechanism is not effective and at best provides the same heating obtained for the resonant mechanism alone. We interpret this feature in the context of the observed stability of Keplerian disks to shear-driven instability, which would suppress the necessary turbulence in our model.

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