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A nonextensive insight into the stellar initial mass function

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Abstract. In the present paper, we propose that the stellar initial mass distributions known as IMF are best fitted by q-Weibull distributions that emerge within nonextensive statistical mechanics. As a result, we show that Salpeter's slope of ~ 2.35 is replaced when a q-Weibull distribution is used. Our results point out that the nonextensive entropic index q represents a new approach for understanding the process of the star-forming and evolution of massive stars.

Resumo. No presente trabalho, propomos que a distribuição inicial de massa estelar conhecida como IMF é melhor ajustada por uma distribuição do tipo q-Weibull que advém da mecânica estatística não extensiva. Como resultado, mostramos que a inclinação da função de Salpeter de ~ 2,35 é recuperada quando uma distribuição q-Weibull é usada. Nossos resultados mostram que o índice entrópico q representa uma nova abordagem para o entendimento do processo de formação e evolução de estrelas massivas.

Keywords. Methods: statistical – Stars: formation

1. Introduction

Almost seventy years after the pio- neering work published by Salpeter (1955), significant progress was made both observationally and theoretically allowing a more accurate description of the initial mass function (IMF).

However, there are still several open theoretical questions, among them, one pointed by Zinnecker (2005). Recently, Cartwright Whitworth (2012) proposed a new IMF description through the stable distributions (e.g.: Gaussian distribution). As mentioned by Maschberger (2013), this kind of distribution considers which the star formation is a purely additive stochastic process. In fact, the IMF scenario is dominated by power-laws. See, for instance, the descriptions proposed by Salpeter (1955) and Kroupa et al. (1993). In particular, Zannetti (2013) analyzed several distributions that can adjust to IMF from lognormal to left truncated beta distributions, as well as the Pareto distribution.

2. $q-$Weibul distribution

Inspired by multifractal systems, Tsallis (1988) proposed the nonextensive entropy S_q, defined by (for further details see Abe & Okamoto (2001))

$$S_q = k\frac{1-\int[p(x)]^q dx}{q-1} \qquad (q \in \Re) \tag{1}$$

where q, denoted as entropic index, is related to the degree of nonextensivity. What is $p(x)$? In this case, the entropy cannot be represented by an additive process, but a non-additive ones denoted by $S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B)$. We can recover the BG entropic additivity if $q = 1$. As cited by [17], the value of this index is a typical characteristic of the system, or a class of universality of the system. A strong property inherent to this generalization is, of course, to assume the Boltzmann exponential as a power-law given by

$$\exp_q(x) = [1 + (1-q)x]_+^{\frac{1}{1-q}} \tag{2}$$

where sign " + " denotes $1 + (1-q)x \geq 0$ and 0 otherwise. For $q \to 1$, the Boltzmann exponential is recovered.

In nonextensive scenario, probability distributions or also called $q-$distributions present important properties that can be used in context of the IMF. To represent these laws in only one distribution function, we need a stretched distribution. In the nonextensive scenario, there is a distribution function denoted by $q-$Weibull that present this behavior, where q denotes the entropic index in statistical mechanics. In this work, we revisit the canonical IMF proposed by Kroupa (2011) and Maschberger (2013). We adopt the nonextensive q-Weibull distributions proposed by Picoli et al. (2003) and recently formulated by Assis et al. (2012) and inspired in Tsallis (1988). The q-Weibull distributions are given by the Probability Density Function:

$$p_q(m) = p_0\frac{r}{m_0}\left(\frac{m}{m_0}\right)^{r-1}\exp\left[-\left(\frac{m}{m_0}\right)^r\right] \tag{3}$$

for $1 + (q-1)(m/m_0)r \geq 0$ and $p_q(m) = 0$ otherwise, where m_0 is a scale parameter and r a shape parameter. Note that for $r = 1$ and $q = 1$ we obtain the $q-$exponential. Respectively, when $q \to 1$ with $r \neq 1$ or $r = 1$ we recover the Weilbull or the exponential distributions. In this context, from this equation above, we can immediately correlate the high mass (hm) index α_{hm} and the nonextensive index q by the relationship

$$\alpha_{hm} = r\left(\frac{2-q}{q-1}\right) + 1. \tag{4}$$

The low mass (lm) index is only associated to the parameter r, in such a way that

$$\alpha_{lm} = r - 1 \tag{5}$$

3. Results

The use of $q-$Weibull distribution function, besides presenting a way of describing the various IMF regimes along the mass

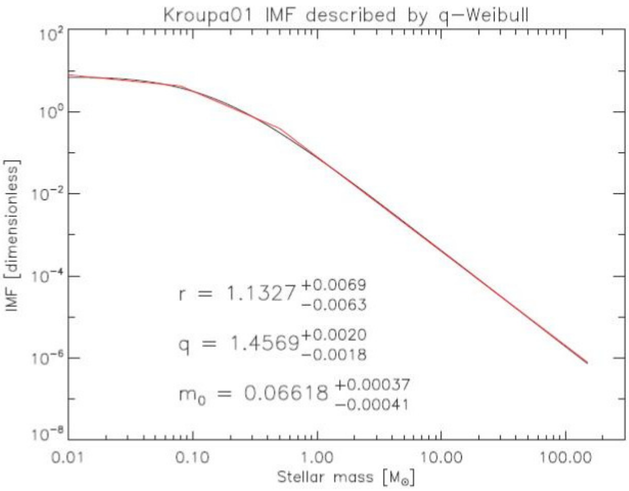

FIGURE 1. Log-log plot of the mass distribution for IMF. The red line represents the best fit using a q-Weibull distribution.

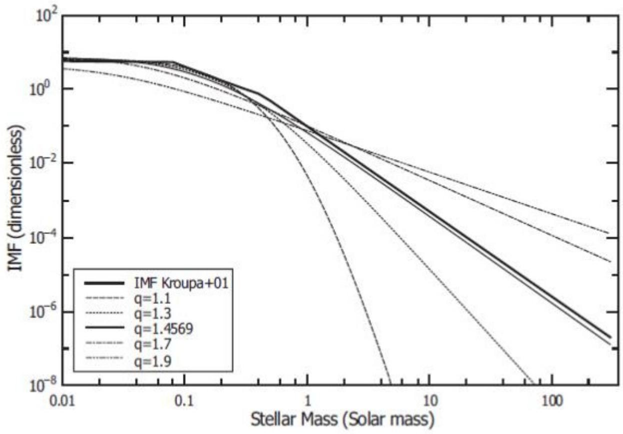

FIGURE 2. Entropic index q influence on the behavior of the q-Weibull distribution with parameters r and m0 fixed and equal to those obtained in the best fit of Fig. 1. The mass distribution for the Kroupa (2011) IMF is represented by the solid, thick line.

bands, has parameters that can adapt to mass functions of systems that present diferent behavior than the one proposed by Kroupa (2011). The value of $q = 1.4569$, obtained by the adjustment of q-Weibull and shown in Figure 1, points to a nonextensivity behavior in the star formation process.

In Figure 2 we have the variation of the index q in the interval between 1 and 2 with step of 0.1 and values of m_0 and r fixed and equal to those obtained in the best fit shown in Figure 1. Also, according to Figure 2, the influence caused by the variation in the values of q for the region of low mass is minimal. That is, it is not this shape parameter, the one responsible for determining the amount of low mass stars that will be born in a certain region, leaving for the r parameter this characteristic. In this context, it is important to remember that the IMF does not determine the number of low-mass stars. On the other hand, in the high mass region, the strong dependence of the $q-$Weibull distribution function on the variation in the values of the entropic index q is clearly evident.

With the passing of generations of stars it is expected that the amount of high mass stars will decrease as opposed to the number of low mass stars that increase. Statistically, the system would tend to steady-state equilibrium ($q = 1$) with the passing of the generations, which would decrease the number of massive stars and, as seen in Figure 2, would lead to a decrease in the q value, making this parameter tend to one. Generally speaking, when the system ages, more massive stars become neutron stars or black holes. In this case, there is a reduction in the number of massive stars in the distribution. As a consequence, the tail of the distribution is reduced which leads the value of q to one, as shown in the curves present in Figure 2. However, the IMF is a probability distribution that takes into account the stars that enter the main sequence, i.e., when the hydrogen ignition is initiated. In this context, the value of computed q must be greater than one, but if we consider the evolution and death of stars that occurs at different rates for different masses, q is a function of time and therefore the future and present-day mass function will affect the behavior of q, as shown in Figure 2.

As a way to help such discussion, let us take the alpha-plots for clusters populations from Kroupa (2011), where it is observed that the scattering of the values become smaller with the increase in the number of bodies in the system, mainly for α_{hm}. It is also observed a shift to the left and slightly above the simulated values for α_{hm} for 70 Myr older clusters when compared to the time at which the IMF was calculated. This behavior may indicate a decrease in the effective number of massive stars as generations pass. Considering the relationship between the α_{hm} values with the entropic index q, this last one can be interpreted as a regulator of the dynamic stellar evolution of massive stars and possibly must be related to age of the star-forming region, or at least as an indication of the number of generations of that region.

4. Conclusion

In our paper, we have investigated the $q-$Weibull distributions which emerge within nonex- tensive statistical mechanics. We verified that it has an upper limit strongly connected with the broadness stellar mass, as defined by equation 4. According to Figure 2, higher values of q are associated to heavy-tails in IMF, whereas lower values of q towards to unity are linked to low-mass distributions. There is also a lower limit of the $q-$Weibull distribution associated to parameter r, as can be seen in equation 5. Besides, our results also point out that the $q-$index can be interpreted as possible indicator of star-forming region age.

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