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Angular momentum conservation for pulsars and core superfluid dynamics

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Abstract. Pulsars are stars that emit radiation in the form of pulses with well-defined frequencies. In the canonical model a pulsar is assumed to be a rotating, highly magnetized sphere made mostly of neutrons. The measurement of the pulse's period allows one to calculate the braking index (n), which informs on the star's rotation deceleration process. One drawback of the canonical model is that for all pulsars it yields n = 3, which does not correspond to observational values. We proposed a theoretical model for pulsars' rotation frequency decay based on the assumption that the star's total moment of inertia would vary with time due to mass motions inside the core. We found that the pulsar J1734-3333 has total angular momentum practically conserved, a phenomenon that we explain relating the motion of neutron superfluid vortices in the core to torques associated to radiation emission. We compared our model to other models that aim at explaining the braking index problem and found them compatible.

Resumo. Pulsares são estrelas que emitem radiação na forma de pulsos com frequências bem definidas. No modelo canônico um pulsar é admitido como uma esfera girante altamente magnetizada feita principalmente de nêutrons. A medida do período do pulso permite que se calcule o índice de frenagem (n), que informa sobre o processo de desaceleração da estrela. Uma desvantagem do modelo canônico é que para todos os pulsares ele fornece n=3,o qual não corresponde aos valores observacionais de n. Nós propusemos um modelo teórico para o decaimento da frequência de rotação de pulsares baseado na suposição de que o momento de inércia total da estrela variaria com o tempo devido a movimento de massas dentro do núcleo. Descobrimos que o pulsar J1734-3333 tem seu momento angular praticamente conservado, um fenômeno que explicamos ao relacionar o movimento de vórtices do superfluido de nêutrons do núcleo com torques associados à emissão de radiação. Comparamos nosso modelo com outros destinados a explicar o problema do índice de frenagem e verificamos que são compatíveis.

Keywords. pulsars: general – pulsars: individual J1734-3333 – stars: interiors

1. Introduction

Neutron stars are formed from the explosion of an ordinary star. With the formation of the neutron star after the explosion of the progenitor star (supernova effect) the matter is in a normal state due the temperatures of the core collapsed.The star cools rapidly through the neutrino irradiation (Manchester & Taylor 1977) and when this temperature falls below Cooper's critical temperature the neutrons form a superfluid (Migdal 1960) and the protons a superconductor (Baym et al. 1969). The electrons in the neutron star are not superconducting because they form a highly relativistic and degenerate plasma and yet, because they are a weakly interacting system, while strong force makes possible the existence of superfluid neutrons and superconducting protons (Baym et al. 1969; Ho & Andersson 2012).

The neutron superfluid presents a sufficient amount of vortex lines parallel to the axis of rotation and therefore on a macroscopic scale the neutrons will be observed with a rotation as a rigid body and soon have a moment of inertia given by the classical theory (Ginzburg & Kirzhnits 1964; Baym et al. 1969).

Normally pulsars are modeled by a rotating highly magnetized sphere which have a magnetic dipole and misaligned with respect to its axis of rotation which is responsible for the observation of emitted radiation in well-defined time intervals in a certain direction, this radiation observed as pulsed radiation is the typical characteristic of these stars (Gold 1968). In such model, which we call canonical, predicts a gradual decrease of the rotation frequency of the star measured by a dimensionless parameter known as braking index represented by n. One common drawback of canonical model is that its magnitude has only one theoretical value for all pulsars, equal to 3, but results derived from observation are different than predicted in the theoretical model and this does not explain the observational values for n (see: Table 1 in (Oliveira et al. 2018)). In a recent paper (Oliveira et al. 2018), we propose a model for pulsars' rotation frequency decay by modifying an assumption of the canonical model.

In this paper (Oliveira et al. 2018) we have assumed that pulsars have a nucleus consisting of superconducting protons and degenerate electrons together with a much larger amount of superfluid neutrons, this nucleus being as large as the crust that is commonly assumed to be thin (Ginzburg 1971; Ho & Andersson 2012; Sourie, Oertel & Novak 2016). As characteristics of pulsars, there is: very high rotations (several rotations per second), very strong fields and with a magnetic dipole with inclination α in relation to its axis of rotation which emits electromagnetic radiation, in well-defined intervals of time, originates from the rotational energy (Pacini 1967; Gold 1968). This paper is organized as follows. In section 2 our model is presented and discussed. In section 3 we discuss the torque found for the pulsar J1734-3333 using our model Section 4 displays our concluding remarks.

2. Angular Momentum

Baym et al. (1969) noted the relationship between changes in the angular momentum of superfluid neutrons and the creation or

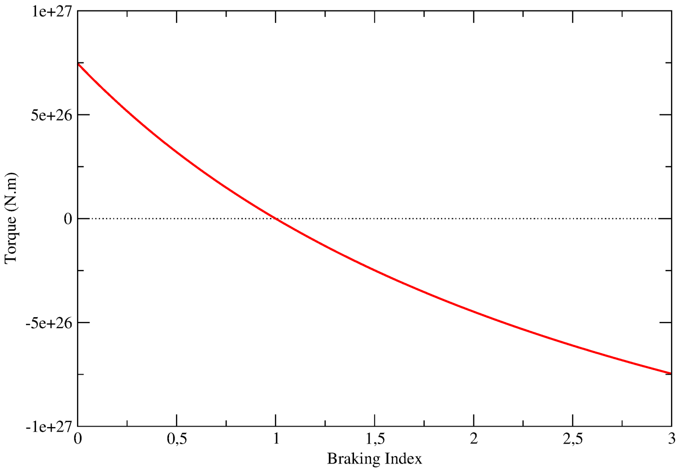

Figure 1. Torque versus braking index for the Pulsar J1734-3333.

destruction of vortices in the superfluid. If the angular velocity is reduced then vortices must be destroyed (Tsuneto 2005). This is consistent with our assumption of the influence of the variation of the displacement parameter on the value of the braking index that could be related to a push given by superfluid vortex lines that continuously hit the inner crust and are then destroyed there as the pulsar spins down. The case $n = 1$ corresponds to total angular momentum conservation, in which the change in angular velocity balances the mass distribution of the superfluid core.

Being the angular momentum proportional to moment of inertia I and angular speed Ω, $L = \Omega I$, and its variation in the time:

$$\dot{L} = \Omega\dot{I} + \dot{\Omega}I. \tag{1}$$

With physical quantity between $\dot{L}$ and L, equal:

$$\frac{\dot{L}}{L} = \frac{\Omega I}{\Omega I}\left(\frac{\dot{I}}{I} + \frac{\dot{\Omega}}{\Omega}\right), \tag{2}$$

where we used $I = 2MR^2/5$ for a sphere,

$$\frac{\dot{I}}{I} = \frac{2\dot{R}}{R} = \left(\frac{\dot{L}}{L} - \frac{\dot{\Omega}}{\Omega}\right). \tag{3}$$

We recall that, rotational kinetic energy is: $E_{rot} = (\Omega^2 I)/2$, with temporal variation: $\dot{E}_{rot} = I\Omega\dot{\Omega} + (\dot{I}\Omega^2)/2$. As above for angular momentum and its derivative:

$$\frac{\dot{E}_{rot}}{E_{rot}} = \frac{I\Omega^2/2}{I\Omega^2/2}\left(\frac{2\dot{\Omega}}{\Omega} + \frac{\dot{I}}{I}\right). \tag{4}$$

with:

$$\frac{\dot{\Omega}}{\Omega} = \frac{\dot{E}_{rot}}{2E_{rot}} - \frac{\dot{R}}{R}. \tag{5}$$

In order to predict the values of braking index, we deduced the following relation in a previous paper (Oliveira et al. 2018):

$$n = \frac{3R\dot{\Omega} + 3\dot{R}\Omega}{R\dot{\Omega} - \dot{R}\Omega} = 3\left(\frac{\dot{\Omega}}{\Omega} + \frac{\dot{R}}{R}\right)\left(\frac{\dot{\Omega}}{\Omega} - \frac{\dot{R}}{R}\right)^{-1}. \tag{6}$$

In this form, we substitute 5 and before 3

$$n = \frac{3(\dot{E}_{rot}/2E_{rot})}{\dot{E}_{rot}/2E_{rot} - (\dot{L}/L - \dot{\Omega}/\Omega)}, \tag{7}$$

and in case of the angular momentum conservation ($\dot{L} = 0$) the equation 1 becomes

$$\dot{L} = 0 = \frac{\dot{\Omega}}{\Omega} + \frac{2\dot{R}}{R} \Rightarrow \frac{\dot{\Omega}}{\Omega} = \frac{\dot{E}_{rot}}{E_{rot}}, \tag{8}$$

and the equation 7:

$$n = \frac{3\dot{E}_{rot}/2E_{rot}}{\dot{E}_{rot}/2E_{rot} + \dot{\Omega}/\Omega} = \frac{3\dot{E}_{rot}/2E_{rot}}{\dot{E}_{rot}/2E_{rot} + \dot{E}_{rot}/E_{rot}} = 1 \tag{9}$$

3. Torque in Pulsar J1734-3333

The total torque can be generally related to the braking index using equation 3 in equation 6, resulting in

$$\dot{L} = \frac{6(n-1)}{5(n+3)}MR^2\dot{\Omega}. \tag{10}$$

From this expression it is straightforward to find that the angular momentum is conserved when $n = 1$.

The behavior of the torque as a function of the braking index is plotted in Figure 1. The value $n = 1$, which corresponds to zero torque or angular momentum conservation, separates two distinct regions: (1) when $n > 1$ the torque is negative and the total angular momentum decays; and (2) when $n < 1$ the torque is positive and the total angular momentum increases.

We interpret the two regions in terms of the balance between the decay in angular momentum due to magnetic dipole radiation emission and the increase in angular momentum due to the increase in due to the increase in moment of inertia as superfluid vortex lines are destroyed: in region (1) dipole radiation would dominate while in region (2) vortex destruction would be dominant. For the small number of pulsars that have braking indices measured with good accuracy so far there are no instances of case (2).

For practical purposes we consider that the pulsar J1734-3333 has $n \sim 1$ (actually 0.9 (±0.2) (Espinoza et al. 2011)). In this case angular momentum is conserved and the torque due to dipole radiation emission is balanced by the torque due to the destruction of superfluid vortex lines that increases the star's moment of inertia.

4. Conclusions

In this paper we have discussed the possibility that when pulsars have decelerated rotation their superfluid cores respond by annihilating the vortex lines that move radially towards the crust. In the particular case of the pulsar J1734-3333 this motion would happen at a rate that nearly equates the amount of angular momentum radiated by the magnetic dipole, resulting in no variation of the crust's angular momentum while the core acts as a reservoir of angular momentum.

A continuation of this study, in which we have previously demonstrated the relationship between the radial flow of superfluid vortex lines and the parameter $\dot{R}$ of our model, is the analysis of an equation of state for the star including superfluity.

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