Boletim da Sociedade Astronômica Brasileira, **31**, no. 1, 33-36

A computational alternative for the polygon method in gravitational microlensing events

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Abstract. In the context of the detection of exoplanets through the technique of gravitational microlensing, one of the most consistent ways of solving the lens equation is to analyze it inversely. This can be obtained by rearranging it in a polynomial of degree $n^2 + 1$ (n, is the number of lenses). Magnification can be obtained with the inverse of the Jacobian for most cases. But the great changes in magnification that occur when the source crosses a caustic is a problem because the Jacobian diverges and we can not use point source solutions or other approximations that depend on the resolution of this equation. The polygon method compute the ratio between the image and source areas but loses precision when we simulate few vertices at the source. In this work, we propose an alternative to the polygon method, which consists of adjusting and adding extra vertices in the 3 images formed by the 5 solutions of the polynomial of degree 5 for the case of 2 lenses, so that the calculation of the magnification is obtained through the ratio between the perimeters of the images, not the areas. In this work, we present the first theoretical results of the application of the referent method.

Resumo. No contexto da detecção de exoplanetas através da técnica de micolentes gravitacionais, uma das formas mais consistentes de resolver a equação da lente, é inverte-la analiticamente. Isto pode ser obtido rearranjando-a em um polinômio de grau $n^2 + 1$ (n, sendo a quatidade de lentes). A magnificação pode ser obtida com o inverso do Jacobiano para a maioria dos casos. Mas, as grandes mudanças na magnificação que ocorrem quando a fonte atravessa uma caustica são um problema pois, o jacobiano diverge e não podemos usar as soluções de fonte pontual ou outras aproximações que dependam da resolução desta equação.O método do polígono, utiliza computa a razão entre as áreas das imagens e da fonte, mas perde precisão quando simulamos poucos vértices na fonte. Neste trabalho, propomos uma alternativa ao método do polígono, que consiste em ajustar e adicionar vértices extras nas 3 imagens formadas pelas 5 soluções do polinômio de grau 5 para o caso de 2 lentes, para que o cálculo da magnificação seja obtido através da relação entre os perímetros das imagens, e não pela área. Neste trabalho, apresentamos os primeiros resultados teóricos da aplicação do referente método.

Keywords. Gravitational lensing: micro – Planets and satellites: detection – Methods: data analysis.

1. Introduction

The gravitational microlensing technique relates the apparent amplification of light from a star in the background to the lens effect created by the passage of another star between the line of sight of the observer and the star in the background. This effect is caused by the deflection of light when passing through objects of great mass. By carefully analyzing the light curves generated by these events, we can detect the presence of exoplanets orbiting such objects. Gravitational microlensing events by themselves had already been detected several times and the potential for exoplanet detection was already known, but it was only in 2003 that the groups Optical Gravitational Lensing Experiment (OGLE) and Microlensing Observations in Astrophysics (MOA) (Bond et al. 2004) confirmed for the first time the presence of a giant planet with a larger half-axis of 4.3 AU around a star of spectral type K in the event OGLE 2003-BLG-235 / MOA-2003-BLG-53 using this technique.

The deflection of the light by a single star can be expressed by $\alpha = \frac{4GM}{c^2 r}$, where α is the deflection angle, M é the lens mass, G is the universal gravitational constant, c is the speed of light and r is the impact parameter. If we establish D_S as the distance between the observer and the source and D_L as the distance between the observer and the lens, we can write the distance between the source and lens as $(D_S - D_L)$, and we can derive the well known equation of the Einstein Radius

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_S - D_L}{D_L D_S}}. \quad (1)$$

If the source and the lens are aligned, we have the so-called Einstein ring. Introducing a apparent distance β between the source and the lens, we can derive the lens equation for the single lens case as $\beta = \theta - \frac{\theta_E^2}{\theta}$ which is known as the lens equation for the single lens case.

1.1. Formalism

For the binary-lens case, we can rewrite β, using the complex notation to denote the lens equation for the two lenses (Witt H. J 1990; Witt & Mao 1995) case, representing a host star and their planet as

$$\omega = z - \frac{\varepsilon_1}{\bar{z} + \bar{z}_1} - \frac{\varepsilon_2}{\bar{z} + \bar{z}_2}. \quad (2)$$

Where, ε_1 and ε_2 are the normalized lenses masses, with $\varepsilon_1 + \varepsilon_2 = 1$. The parameter z is the two-dimensional position written as the real and imaginary components of a complex number. The ω is the relative position of the source at a specific time. The bar over complex quantities indicates complex conjugation.

To solve the lens equation with $n = 2$, it is necessary to invert a 5th order polynomial and solve it to find the polynomial roots as shown by Witt H. J (1990). For the case where the source is not close enough to the caustic-crossing region, we can use the point source magnification method to solve and obtain the light curve. The lens equation can be represented by the folown:

$$\sum_{l=0}^{n^2+1} c_l z^l = 0. \quad (3)$$

This is the polynomial form of the lens equation for the case of n lenses. The coefficients of this general polynomial are given as:

$$c_l = \sum_{i=0}^{n} (\eta_{i,l-1} X_i - \eta_{i,l} W_i) \quad (4)$$

Where l varies from 0 to $(n^2 + 1)$.

We can now use the polynomial form of the lens equation seen in the equation 3 for the case of a two lenses system. In this case, the polynomial is of the fifth degree ($5 = n^2 + 1$) and the equation of the lens is in the form

$$\sum_{l=0}^{5} c_l z^l = 0 \Rightarrow c_0 + c_1 z^1 + c_2 z^2 + c_3 z^3 + c_4 z^4 + c_5 z^5 = 0. \quad (5)$$

And their coefficients will be given according to equation:

$$c_l = \eta_{0,l-1} X_0 - \eta_{1,l-1} X_1 + \eta_{2,l-1} X_2 - [\eta_{0,l} W_0 - \eta_{1,l} W_1 + \eta_{2,l} W_2] \quad (6)$$

From vector calculus, we know that if an infinitesimal area $dsd\theta$ is mapped in the source plane, its area projected $drd\theta$ can be calculated through the Jacobian determinant. The Jacobian specifies the change in an infinitesimal area when passing through a given transformation. The Jacobian returns the ratio between an element of infinitesimal area in the plane of the source divided by its corresponding area in the plane of the image, thus, to find the magnification, which is the ratio between the infinitesimal area of the image and its source generator element, we need to compute the inverse of the Jacobian,

$$M_T = \sum_I \frac{1}{|J_i|} = \frac{1}{|J_+|} + \frac{1}{|J_-|}. \quad (7)$$

But the high changes in magnification that occur when the source crosses the caustic can be a problem, because the Jacobian diverges and we cannot use point source solutions or other approximations that depend on the resolution of this equation. In this way, another alternative must be investigated. The polygon method, which is based on the Stokes theorem (Gould & Gaucherel 1997), can be written as follows, it is an algorithm that aims to get around this problem by making the source disk approximate a polygon,in this way,the equation of the lens can be solved for each vertex producing the images also formed by polygons.The areas of these polygons are then evaluated analytically and the magnification can be approximated by the ratio between the areas of the images and the area of the source.

2. The ratio of perimeters

Our method consists of a geometrical/computational procedure, that instead of computing the areas of the polygon that form the images, we directly compute the sum of the distances between the points of the images, and thus we can make the fraction between the perimeters. In addition, we add extra vertices to the source so that the 3 or 5 images formed have relatively equivalent points for calculating the perimeters. Since we do not need to calculate the areas, the error related to the limited number of vertices is smaller (as we going to demonstrate). If we compare the area of the square with the area of the circle, we have a relative error in the value that depends on the number of vertices. The more vertices, the smaller the error. However, if we compute the perimeters of the square and the circle, the relative error starts smaller. As we are interested in the ratio of areas, it is the same as the ratio of perimeters. Using the perimeters rather than the areas, we can represent the images by polygons with fewer vertices and obtain equal accuracy.

The area of a regular polygon is given by:

$$A_{rp} = \frac{nla}{2}, \quad (8)$$

where n is the number of vertices, l is the length between two consecutive vertices and a is the apothem of one of the triangles that form the regular polygon. Since we want to know the area of this polygon with respect to a circle of radius r, we can parameterize the equations in relation to this radius. Following the geometry of the problem, we get:

$$a = r\cos(\frac{\pi}{n}), \quad (9)$$

thus, the polygon area is:

$$A_{rp} = (r\cos\frac{\pi}{n})^2 n \tan\frac{\pi}{n} \quad (10)$$

By dividing this value by the area of the circle that comprises the radius previously used and simplifying the equation, we have:

$$A_{c/p} = n\frac{\sin\frac{2\pi}{n}}{(2\pi)} \quad (11)$$

The equation 11 represents the degree of confidence between the area of the polygon of n vertices and the area of a circle of radius r. Using the above equations we can then calculate the length of each polygon edge, and then compute its perimeter.

$$P_{pr} = 2na\tan\frac{\pi}{nv} \quad (12)$$

By dividing the equation 12 by the perimeter of the circle, which is $2\pi r$, we have the degree of confidence between the perimeter of the polygon and the circumference that results in our final equation:

$$P_{c/p} = an\frac{\tan\frac{\pi}{n}}{pir} \quad (13)$$

The figure 1 demonstrates how the precision of the equations 11 and 13 varies with the number of vertices used. It is evident that to do the ratio of sizes between the perimeters is more precise of doing the ratio of the sizes of the areas for the case of few vertices. By increasing the number of vertices, the perimeter of the polygon approaches the perimeter of the circle, and then we have that the areas will be equal.

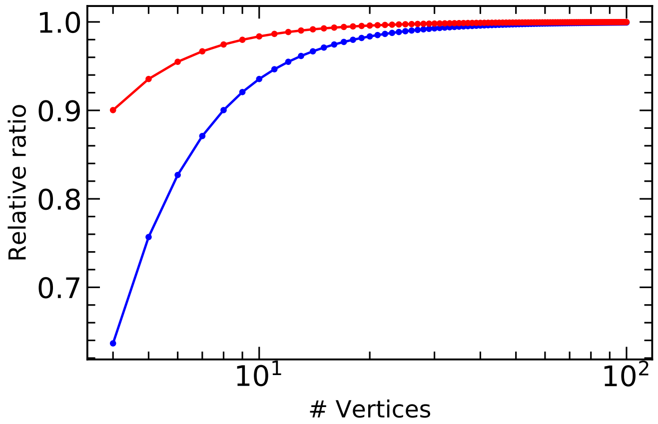

Figure 1. Blue line: $A_{c/p}$ with number of vertices ranging from 4 to 100. Red line $P_{c/p}$ with same range of vertices.

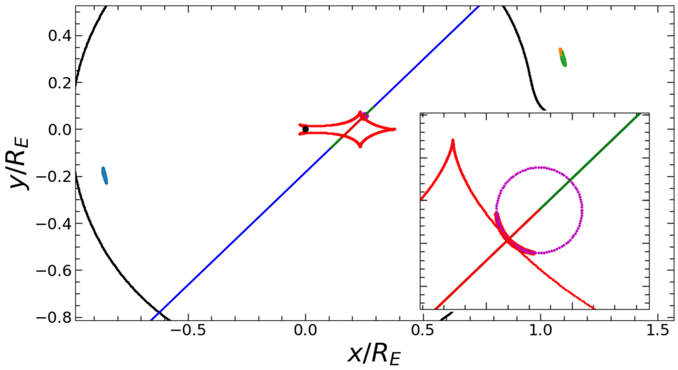

Figure 2. The red line represents the caustic for a resonant system, the colors green, orange, and light blue represent the solutions of the lens equation. The zoom frame shows the accumulation of points in the source representation by our method. The green line shows when the method starts to be used.

Now that we know that ratio between the perimeters is more precise than the ratio between the areas, we can do the distribution of points in the polygon in order to optimize the amount of equally spaced points in the images when the source crosses some caustic. This is done inversely by measuring the distance between the points in the images and by regulating the number of vertices in the source creation. Then more vertices are created when the source approaches the set limit for the use of hexadecapole approximation (Gould 2008).

Figure 2 shows the decision process between using the point source method, Hexadecapole approach (Andrew Gould 2008) or the alternative proposed in this work. As the edges of source approach the caustic, the method used to calculate the magnification changes. Figure 3 shows the difference in the resolution of the image construction using the proposed method or not. We see that, if we compute only the finite source as an equally distributed disk, the images are not completed in the 5 solutions of the lens equation when the source crosses the caustic. If we use our method, the images are better represented and so we can compute more safely the perimeter of them to reach the magnification of the finite source.

3. Summary and conclusions

We have shown that the calculation of the relative magnification of a gravitational microlensing event can better be approximated by the ratio of the perimeters of the images to the source, rather than the ratio of the areas. We conclude that for a low number of vertices (ie minimum of 5 vertices) the perimeter ratio is 2 times more accurate than the ratio of the areas. We do not yet apply these equations to real data, but the method has the potential to accelerate the modeling of light curves since it depends on fewer vertices to reach the same precision as the ratio between areas. Second order effects such as Limb Darkening can easily be included in this method in the same way as in the polygon method.

Acknowledgements. We thank the University of Rio Grande do Norte and CAPES for the financial support.

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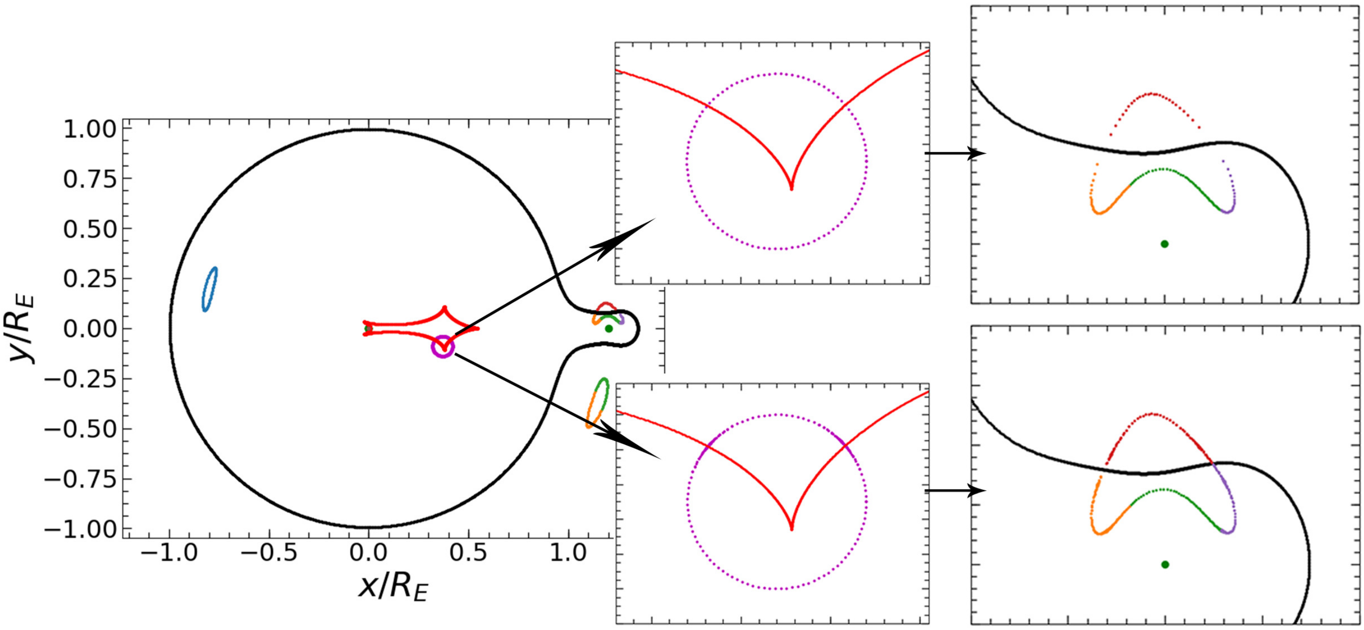

Figure 3. The first frame shows the gravitational microlensing event topology with $q = 3E-3$ and $s = 1.3$. The different colors represent the solutions of the lens equation for that instant. The top frames show the source and the formation of one of the images using equally spaced points. The below frames show the arrangement of the source points and the formation of the images by using our method.