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On the multifractality in GW150914 gravitational wave

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Abstract. We analyze the data of the GW150914 gravitational wave signals detected by LIGO through the lens of multifractal formalism using the MFDMA method, as well as shuffled and surrogate procedures. We identified two regimes of multifractality in the strain measure of the time serie by examining long memory and the presence of nonlinearities. The moment used to divide the serie into two parts separates these two regimes and can be interpreted as the moment of collision between the black holes.

Resumo. Analisamos os dados do sinal de onda gravitacional GW150914 detectado pelo LIGO a partir do ponto de vista do formalismo multifractal usando o método MFDMA, bem como os procedimentos *shuffled e surrogate. Identificamos dois regimes de multifractalidade nas medidas da série temporal de deformação relativa examinando memória de longo prazo e presença de não linearidades. O momento usado para dividir a série em duas partes separam estes dois regimes e pode ser interpretado como o momemnto da colisão entre os buracos negros.*

Keywords. Gravitational waves – Multifractal analysis

1. Introduction

In February 11, 2016, data on the first direct observation of gravitational waves (hereafter, GWs) from a binary black hole merger was published, opening up new avenues for studying the universe beyond the analysis of electromagnetic waves (Abbott et al. 2016a; Aasi et al. 2015; Abbott et al. 2016c). The GW150914 signal was generated from the merger of two black holes with masses of $29^{+4}_{-4}M_{\odot}$ and $36^{+5}_{-4}M_{\odot}$. The event was observed by the two advanced LIGO (Aasi et al. 2015) detectors with a statistical significance of 5.1σ. The GW150914 data used here are within the range of 32 seconds around the event and have a measurement frequency of 4096Hz and will be considered a time-seres of strain measures.

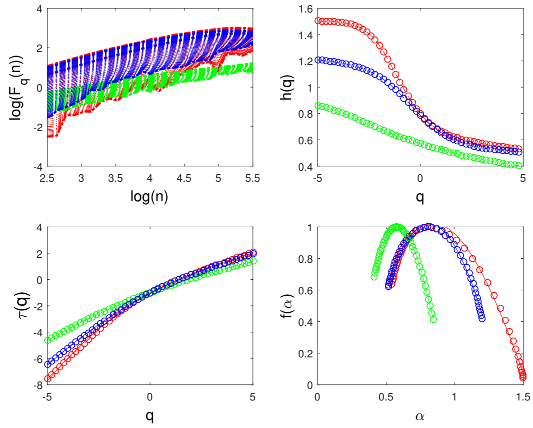

Figure 1. Multifractal analysis of the full data from GW150914 (`H1data`). The top-left panel shows the fluctuation function versus the multi-scale behavior in a log-log diagram. The original series is in red, the shuffled series is in green, and the upper and lower limits correspond to $q = 5$ and $q = -5$, respectively, while the bold in the middle corresponds to $q = 0$. Dependences on the q_{th} moment of the generalized Hurst exponent, h(q), and the multifractal scaling exponent, $\tau(q)$, are shown in the top-right and bottom-left panels, respectively. The multifractal spectrum is shown in the bottom-right panel.

2. Multifractal analysis

Since the end of the last century, several works, including de Freitas et al. (2016, 2017); Mali (2016); Norouzzadeh et al. (2007); Tanna & Pathak (2014), have applied multifractal analyses to time series. MFDMA with a backward-moving average ($\theta = 0$) has been found to yield parameters with better alignments with the numerically calculated parameters, and this is the method employed in this work. Additionally, this value of θ has been demonstrated to achieve the best performance (Eghdami et al. 2017; de Freitas et al. 2017; Gu & Zhou 2010). Tha analysis presented here is in accordance with the MFDMA procedure described by Gu & Zhou (2010).

3. Results and discussion

3.1. Shufled and surrogate procedure

To investigate each type of multifractality, two methods, the shuffling and surrogate methods, are commonly used. These methods essentially consist of modifying the original sample to eliminate the source of the multifractality. The shuffling method randomizes the position of the data in the series to destroy the memory of the series and hence the long-term correlation. Applying an MFDMA to shuffled series will result in monofractal behavior, i.e., $h(2) \sim 0.5$ and $\Delta\alpha \sim 0$, if the series presents multifractality only due to long-term correlations. This procedure does not affect the other source of multifractality. Separately, the surrogate procedure generates a series via a Fourier transform, preserving amplitudes but randomizing the phases, and then performs an inverse Fourier transform. This eliminates non-linearities in the series but does not affect the long-range correlations; thus, the generalized Hurst exponent of

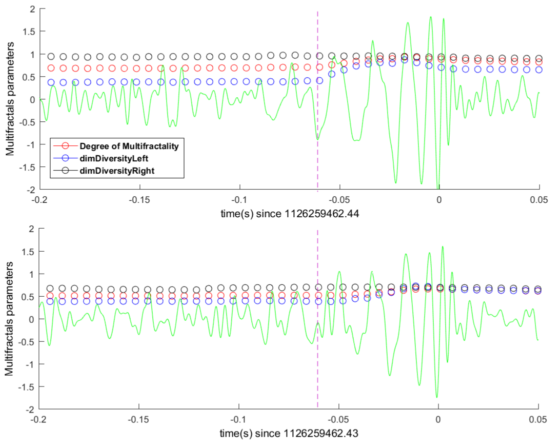

Figure 2. Point-to-point multifractal analysis for the GW150914 time series from Livingston (bottom panel) and Hanford (top panel; shifted and inverted (Abbott et al. 2016a)), illustrated in green. Red circles represent the degree of multifractality ($\Delta\alpha$) calculated in the time series up to that point; likewise, blue and black circles represent the left side diversity $f(\alpha)_{max} - f(\alpha)_{min}^{left}$ and right side diversity $f(\alpha)_{max} - f(\alpha)_{min}^{right}$, respectively. The vertical lines represent $t = -0.06s$, the time point at which the time series are divided.

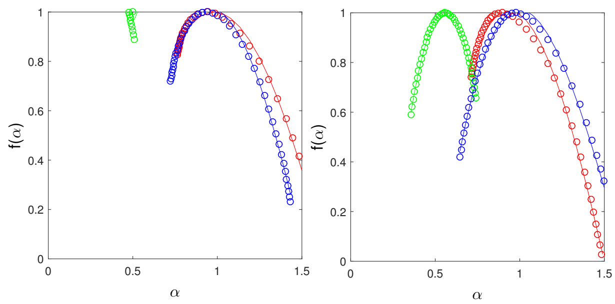

Figure 3. The multifractal spectrum of the first (left panel) and second (right panel) parts of the GW150914 time series are in red. The results of the application of the shuffled and surrogate methods are in green and blue, respectively.

surrogate series will not exhibit a dependence on q. The results of the MFDMA application in GW1509814 data are presented in Figure 1 and point to a multifractal behavior for this time-series. Neither shuffled nor surrogate procedures can be destroy the multifractality, indicating that the two sources are present.

The investigation point-to-point shown in Figure 2 indicate an elevation on left side diversity parameter. The division of the division of the original series into two sub-series at this point allows identification of two multifractals regimes indicated in Figure 3. In this figure we can see the multifractal spectra for the first (left panel) and second (right panel) parts of the orginal series. The shuffled procedure succeeds in destroying the multifractality in the first part, but not in the second. Therefore, both types are present in the second part.

We can associate the memory present in the oscillatory behavior as being the of long-term memory. In the second part, this behavior remains, but it is accompanied by a widening in the tails of the distribution of the strain measurements due to the increase in the gravitational wave amplitude. As this increase only occurs due to the collision of the black holes, the time used to divide the sub-series can be interpreted as the beginning of the merger phase.

4. Conclusions

The statistical approach proposed in this study highlights the scenario opened by detection of the first GWs. We summarize the main results in three points: i) characterize the fractal dynamics of the signals, identifying their multifractal sources; ii) find the moment of the beginning of merger phase in black hole coalescence system, and; iii) determine the empirical relationship between the variation in left side diversity and chirp mass as an additional way for estimating this latter parameter. The methodology applied here may serve as a standard procedure for future analyses of gravitational waves.

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