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Bayesian correction of $H(z)$ data uncertainties

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Abstract. We compile 41 $H(z)$ data from literature and use them to constrain OΛCDM and flat ΛCDM parameters. We show that the available $H(z)$ suffers from uncertainties overestimation and propose a Bayesian method to reduce them. As a result of this method, using $H(z)$ only, we find, in the context of OΛCDM, $H_0 = 69.5 \pm 2.5\,\mathrm{km\,s^{-1}Mpc^{-1}}$, $\Omega_m = 0.242 \pm 0.036$ and $\Omega_\Lambda = 0.68 \pm 0.14$. In the context of flat ΛCDM model, we have found $H_0 = 70.4 \pm 1.2\,\mathrm{km\,s^{-1}Mpc^{-1}}$ and $\Omega_m = 0.256 \pm 0.014$. This corresponds to an uncertainty reduction of up to ≈ 30% when compared to the uncorrected analysis in both cases.

Resumo. Compilamos 41 dados de $H(z)$ da literatura e os usamos para vincular os parâmetros de OΛCDM e ΛCDM. Mostramos que os dados de $H(z)$ disponíveis sofrem de superestimação das incertezas e propomos um método bayesiano para reduzi-las. Como resultado, usando apenas $H(z)$, encontramos, no contexto do OΛCDM, $H_0 = 69,5 \pm 2,5\,\mathrm{km\,s^{-1}Mpc^{-1}}$, $\Omega_m = 0,242 \pm 0,036$ e $\Omega_\Lambda = 0,68 \pm 0,14$. No contexto do modelo ΛCDM plano, encontramos $H_0 = 70,4 \pm 1,2\,\mathrm{km\,s^{-1}Mpc^{-1}}$ e $\Omega_m = 0,256 \pm 0,014$. Isso corresponde a uma redução das incertezas de até ≈ 30% em comparação com a análise sem correção em ambos os casos.

Keywords. Cosmology: observations – dark energy – dark matter

1. Cosmic Dynamics of ΛCDM Model

By considering an homogeneous and isotropic FRW geometry (with $c = 1$) and inserting it in the Einstein Field Equations, with baryons, cold dark matter and cosmological constant, we may write for the expansion rate

$$H(z) = H_0\left[\Omega_m(1+z)^3 + (1-\Omega_m-\Omega_\Lambda)(1+z)^2 + \Omega_\Lambda\right]^{\frac{1}{2}} \tag{1}$$

where $\Omega_i = \frac{\rho_i}{\rho_c}$ are parameter densities and we have used the normalization condition $\Omega_m + \Omega_\Lambda + \Omega_k = 1$. The standard, concordance flat ΛCDM model has $\Omega_k = 0$.

2. $H(z)$ data

Hubble parameter data are inferred from differential ages, BAO and LRG, not depending on any background cosmological models. The data we work here are a combination of two compilations: Sharov & Vorontsova (2014) and Moresco et al. (2016). By combining both datasets, we arrive at 41 $H(z)$ data. From these data, we perform a χ^2 analysis by minimizing the χ^2 function of free parameters:

$$\chi^2 = \sum_{i=1}^{41}\left[\frac{H_0E(z_i,\Omega_m,\Omega_\Lambda) - H_i}{\sigma_{Hi}}\right]^2 \tag{2}$$

where $E(z) \equiv \frac{H(z)}{H_0}$ and $H(z)$ is given by Eq. (1).

3. Data analysis and goodness of fit

In order to find the constraints over the free parameters $(H_0, \Omega_m, \Omega_\Lambda)$, we have sampled the likelihood $\mathscr{L} \propto e^{-\chi^2/2}$ through Monte Carlo Markov Chain (MCMC) analysis. We have used the so called Affine Invariant MCMC Ensemble Sampler by Goodman & Weare (2010), which was implemented in Python language with the emcee software by Foreman-Mackey et al. (2013). From this analysis, we find the red confidence contours on Fig. 1 and find $\chi^2_\nu \equiv \chi^2_{min}/\nu = 0.488$ for OΛCDM.

As it is well known (Bevington & Robinson 2003; Vuolo 1996), the expected value of χ^2_ν from its probability distribution is $\chi^2_\nu = 1$. Values very far from this are unlikely. High χ^2_ν values may indicate underestimation of uncertainties or poor fitting of the model, while low values of χ^2_ν indicate, in general, overestimation of uncertainties.

We calculate the cdf for χ^2_ν and find the probability of obtaining χ^2_ν as low as 0.488 for $\nu = 38$, $P(\chi^2_\nu < 0.488) = 0.3342\%$. It indicates a very low and unlikely χ^2 value, which, in turn, from Eq. (2) indicates overestimated $H(z)$ uncertainties.

4. $H(z)$ uncertainties correction

Following Hogg et al. (2010), we have considered the likelihood as a generative model for the data. It is a parameterized statistical procedure to reasonably generate the given data. Here, we consider the σ_i to be all overestimated by a constant factor f, thus, $\sigma_{i,true} = f\sigma_i$. For OΛCDM, then, our set of free parameters now is $\theta = (H_0, \Omega_m, \Omega_\Lambda, f)$, to be constrained from the likelihood:

$$\ln\mathscr{L} = -\frac{1}{2}\sum_{i=1}^{n}\left\{\frac{[H_i - H(z_i,H_0,\Omega_m,\Omega_\Lambda)]^2}{f^2\sigma^2_{Hi}} + \ln(2\pi f^2\sigma^2_{Hi})\right\} \tag{3}$$

By doing the same procedure of last section, now with the additional parameter f, we find the constraints shown on Fig. 1 (black lines).

From Fig. 1, we may see the difference in the parameter space when we introduce the f parameter. The parameter constraints are shown on Table 1.

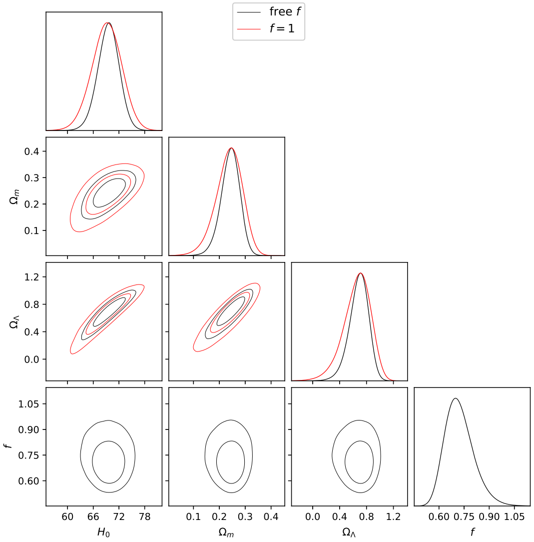

Figure 1. OΛCDM model, with contours for 68.3% and 95.4% confidence intervals. H_0 in km/s/Mpc.

Table 1. Mean values of parameters of OΛCDM model from $H(z)$ data, without uncertainties correction and with uncertainties correction factor f. Uncertainties correspond to 68% c.l.

Parameter	Uncorrected	Corrected
H_0	69.1 ± 3.5	69.5 ± 2.5
Ω_m	0.237 ± 0.051	0.242 ± 0.036
Ω_Λ	0.66 ± 0.20	0.68 ± 0.14
f	–	$0.723^{+0.084}_{-0.085}$

On Table 1, we may see the parameters uncertainties reduction. An interesting feature we may see from Fig. 1 is that the f parameter is much uncorrelated to cosmological parameters.

4.1. Flat ΛCDM

We have also analysed flat ΛCDM model. The results of this analysis may be seen on Fig. 2 and Table 2.

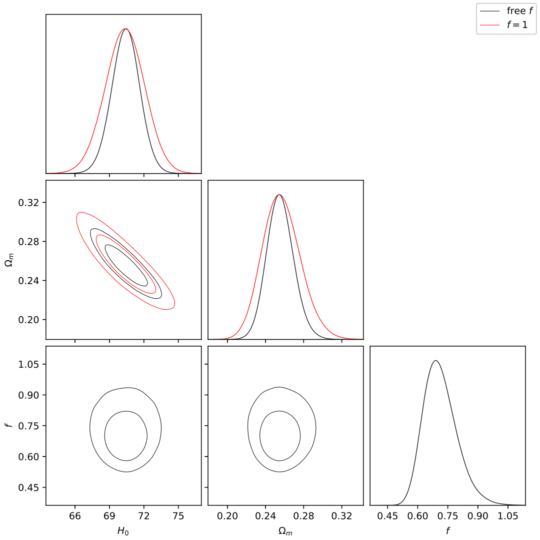

Figure 2. Flat ΛCDM model, with contours for 68.3% and 95.4% confidence intervals. H_0 in km/s/Mpc.

Table 2. Mean values of parameters of flat ΛCDM model from $H(z)$ data, without uncertainties correction and with uncertainties correction factor f. Uncertainties correspond to 68% c.l.

Parameter	Uncorrected	Corrected
H_0	70.3 ± 1.7	70.4 ± 1.2
Ω_m	0.257 ± 0.020	0.256 ± 0.014
f	–	0.714 ± 0.082

The H_0 uncertainty is reduced from 1.7 to 1.2, Ω_m uncertainty has reduced from 0.020 to 0.014.

5. Discussion and Conclusion

With 34 $H(z)$ data, Sharov & Vorontsova (2014) find a more stringent result, namely, $H_0 = 70.26 \pm 0.32$, $\Omega_m = 0.276^{+0.009}_{-0.008}$ and $\Omega_\Lambda = 0.769 \pm 0.029$. However, they have combined $H(z)$ data with SNe Ia and BAO data, which is beyond the scope of our present work, but our value is compatible at 1σ c.l.

Moresco *et al.* have used their compilation of 30 $H(z)$+H_0 to constrain the transition redshift from deceleration to acceleration, in the context of ΛCDM $\left(z_t = \left[\frac{2\Omega_\Lambda}{\Omega_m}\right]^{1/3} - 1\right)$. They have found $z_t = 0.64^{+0.11}_{-0.07}$. By using our 41 $H(z)$ data, we find $z_t = 0.77 \pm 0.22$ without correction and $z_t = 0.78 \pm 0.15$ with the f correction.

We have developed a promising method to reestimate uncertainties.

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