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Application of the restricted problem of the three bodies in teaching physics and astronomy

G. S. Macedo & A. J. Roberto Jr.

[1] Federal University of Alfenas, e-mail: gabrielmacedo154@yahoo.com.br, arturjustiniano@gmail.com

Abstract. In this work, we will present a construction of a small code, which aims to numerically solve the restricted problem of the three bodies. Theme with numerous publications that lodge methods to solve. Here we go beyond solving them apply them in two systems, Sol-Jupiter-Trojan Jupiter and Sol-Neptune-Pluto.

Resumo. Neste trabalho vamos apresentar a construção de um pequeno código, que visa resolver numericamente o problema restrito dos três corpos. Tema com inúmeras publicações que apresentam metodos de como resolver. Aqui vamos alem de resolver aplica-los em dois sistema, Sol-Jupiter-Troiano Jupiteriano e Sol-Netuno-Plutão.

Keywords. Celestial mechanics – Teaching of Astronomy – Chaos

1. Introduction

Isaac Newton (1643-1727) when he published his book: "Philosophiae naturalis principia mathematica", also referred to as principia, wrote the laws of the motion and the law of universal gravitation and also showed the main problem of celestial mechanics, the problem of n bodies (Prazeres 2010). The problem of n bodies can be described as, given to a system with n point masses, that is, points of mass that have their movements governed by gravitation and also given their initial conditions, their position and its speed, what is desired is to know its position and velocity moment future. Mathematically translating is to solve the Ordinary Differential Equations (ODE's) which govern the movement of the body in question. On a system with n bodies there are multiple near collisions make the problem extremely complex, which even present moment, there is no analytical solution (Prado 2001). For n> 2 there is no analytical solution to the problem, but a particular case of the three-body problem (n = 3) has, is the denominated the Three-Body Restricted Problem (PRTC).

The PRTC is a theme with numerous publications in magazines specialized and some in the area of physical education and astronomy. However, in relation to the latter we do not find in the publications application examples of this problem, only the development of methods for solve the related ODE's and it. What for us is a misunderstanding, since the PRTC is an as- directly related to the contents of the subjects of classical mechanics, astronomy and numerical methods and which has great potential to contextualize the study content. Thinking about that, in this work we go present a teaching strategy that can be used to include the PRTC in teaching physics and astronomy. It is, development of a small numerical code for to solve the ODE's of the PRTC by the Cauchy method and graphically simulate the solution found. To exemplify and contextualize we apply in two cases. The system Jupiter, one of its Trojan asteroids and the Sun and the other composed by the Sun, Neptune and Pluto.

2. Restricted problem of the three bodies

Euler in 1772, in his book "Theoria motuum lunae, new methodo pertractata ", proposed a reformulation for the problem of three bodies. This particular case is a version simplified from the three-body problem and became the studied in celestial mechanics due to its applications and was later named by Henri Poincaré (1854 - 1912) of Restricted Problem of the Three Bodies (Prazeres 2010).

Such a problem can be described as follows, given three mass bodies m_1, m_2 (called primary) and m_3 that move under mutual gravitational action and knowing their initial conditions, position and velocity, desires to determine their subsequent motions (Prado 2001).

The three-body problem has no analytical solution due to unsurpassed mathematical complications, but a particular case it has solution and multiple applications this and the Restricted Problem of Three Bodies. There are several cases of PRTC, but we will be concerned about the problem restricted circular plane of the three bodies. Hellings (1994) describes the simplifications made in the problem of three bodies which are:

1. The mass of the third body is practically zero, that the third body has no in the orbits of the primary.

2. The orbits of the circular primaries.

3. The third body moves in the orbital plane of the primaries. This restriction implies an important simplification mathematics, since now the complete description of the prob- lems becomes two-dimensional.

3. Methodological Procedures

The equation that governs the movement of bodies is the Law of Universal Gravitation, this is the most important equation because all others derive from it. Based on the declarations 1,2 3 we can discard equations of motion of the primary staying only with the equation of the third body and equating Newton's second Law with the Law of Universal gravitation and using the coordinate system dimensionless to simplify the equations of motion as described by Hellings (1994), we arrive at the equations that govern the movement of the third body.

$$x'' = -(1-\mu)\frac{x-\mu}{R_1^3} - \mu\frac{x+1-\mu}{R_2^3} + x + 2y' \qquad (1)$$

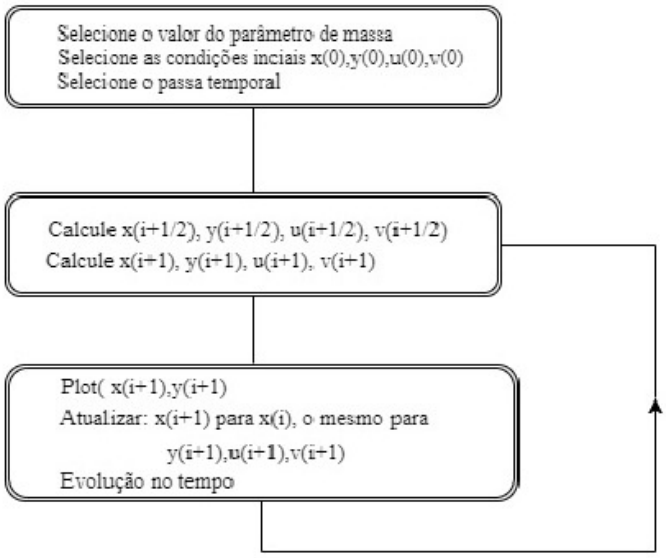

Figure 1. Cauchy numerical method

$$y'' = -(1-\mu)\frac{y}{R_1^3} - \mu\frac{y}{R_2^3} + y - 2x' \tag{2}$$

onde

$$R_1 = \sqrt{(x-\mu)^2 + y^2} \tag{3}$$

$$R_2 = \sqrt{(x+1-\mu)^2 + y^2} \tag{4}$$

In order to solve equations 1 and 2 it was de- developed a small algorithm that uses the Cauchy method, being necessary as initial parameters the ratio mass, initial position, x_0 and y_0, and the speeds initials x_0' and y_0'. The method used is described in the figure follow.

4. Results

We apply the algorithm developed in system the Sol–Jupiter–Asteroid and we managed to trace the periodic orbit of a Jupiterian asteroid, see Fig. 2.

And also in the Sun–Netuno–Pluto system we to calculate the Pluto's libration due to Neptune, as shows 3.

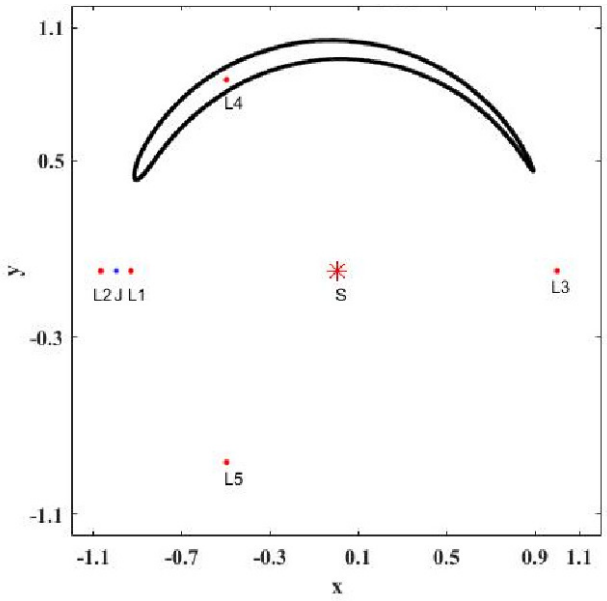

Figure 2. Orbit of a Jupiterian Trojan asteroid

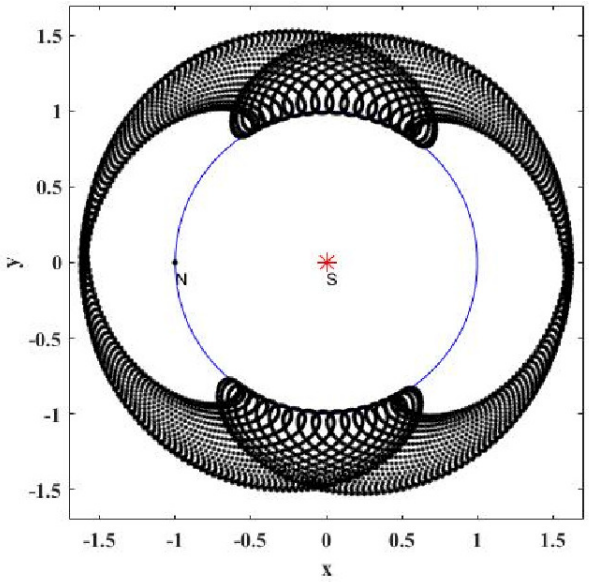

Figure 3. Pluto Orbit disturbed by Neptune

5. Conclusion

With the results we noticed that the present work has a potential in the teaching of physics and astronomy, aiming mainly at the dynamics of celestial bodies. Therefore, our project is to transfer this code to a more attractive graphical platform to be used as a teaching tool and the construction of a teaching sequence that can be applied in the disciplines of physics and astronomy.

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