Boletim da Sociedade Astronômica Brasileira, **30**, no. 1, 138-141

Stellar ages: combination of the Kinematical and Isochronal methods

F. Almeida-Fernandes[1] & H. J. Rocha-Pinto[2]

[1] Observatório do Valongo, Universidade Federal do Rio de Janeiro
e-mail: `felipe11@astro.ufrj.br,felipefer42@gmail.com`

Abstract. We have developed a method to estimate a probability density function (pdf) for the age of a star based on its spatial velocity. The fundaments of the method is the formalism of the velocity ellipsoid and the increase of the dispersion of stellar velocities with age. We show how this method can be used to infer properties of interest to study the evolution of the Galaxy, like the star formation history and the age-metallicity relation. We explain how the age pdf obtained by the kinematical method can be used as a prior pdf for the isochronal method, resulting in better constrained ages. In order to implement the isochronal method, we developed a python library to perform the calculations involving isochrones and evolutionary tracks: the EITApy code. To quantify the benefits of combining both methods, we simulated a sample of 10000 stars with known ages and analysed the results for different regions of the HR diagram. We show that for some groups, age uncertainties are 20% smaller. As an application of the method, we obtained the ages for the stars of the S4N survey and obtained its kinematical parameters using the orbital integrator galpy. The next step is to investigate how the relations between compositions and kinematical parameters with age can be explained in the context of the chemodynamical evolution of the Galaxy.

Resumo. Desenvolvemos um método para estimar uma função de densidade de probabilidade (pdf) para a idade de uma estrela com base em sua velocidade espacial. O método se fundamenta no formalismo do elipsoide de velocidades e no aumento da dispersão de velocidades com a idade. Mostramos que o método pode ser utilizado para inferir propriedades de interesse para o estudo da evolução da Galáxia, como o histórico de formação estelar e a relação idade–metalicidade. Descrevemos como a pdf de idade obtida pelo método cinemático pode ser utilizada como distribuição prior no método isocronal, resultando em idades melhor definidas. Para aplicar o método isocronal, desenvolvemos uma biblioteca em python que realiza os calculos numéricos envolvendo as isócronas e trajetórias evolutivas: o código EITApy. Para quantificar os benefícios da combinação dos dois métodos, simulamos uma amostra de 10000 estrelas com idades conhecidades e analisamos os resultados para diferentes regiões do diagrama HR. Mostramos que para alguns grupos, as incertezas são 20% menores. Aplicamos os métodos para as estrelas do levantamento S^{4}N, além de obter seus parâmetros cinemáticos utilizando o integrador orbital galpy. O próximo passo, será investigar como as relações entre composição química e parâmetros cinemáticos com a idade podem ser explicadas em um cenário de evolução quimiodinâmica da Galáxia.

Keywords. Stars: evolution – Stars: kinematics and dynamics – Stars: statistics

1. Introduction

The stellar composition and kinematics are a reflex of the interstellar medium properties at a given epoch, and the age allows us to place these properties in time. Ages are derived from properties that change with time in a way that can be predicted empirically and/or theoretically.

For studies that aim to understand the chemodynamical evolution of the Galaxy, it's of great importance to be able to derive precise ages for long lived stars. It's these stars that have been around since the formation of the Galaxy and will contain key informations about its state in different epochs. But when working with long lived stars, a new problem arises: these stars live this long because their internal evolution is slower, therefore, their observation properties change in slower rate, difficulting the inference of the ages.

In this work, we developed a method to derive a stellar age that is independent from its internal evolution, the Kinematical Method. We combined this method with the Isochronal Method, which consists of comparing the observed atmospheric parameters with those predicted by evolutionary models. We showed that it allows us to derive better constrained ages for late-type stars, which we expect will contribute to provide more data to understand the Galactic evolution. We also build the python module EITApy for the numerical calculations.

2. The Kinematical Method

The velocity dispersion of a group of stars increases with the age of the stars (Nordström et al. 2004). It causes the velocitie's probabilities to depend on age.

The Bayes theorem allows us to reverse this probability dependence and obtain the probability for the age, given the velocities. To simplify the analysis we use the formalism of the velocity ellipsoid:

$$v_1 = (U + U_\odot)\cos\ell_v + (V + V'_\odot)\sin\ell_v\,, \tag{1a}$$
$$v_2 = -(U + U_\odot)\sin\ell_v + (V + V'_\odot)\cos\ell_v\,, \tag{1b}$$
$$v_3 = W + W_\odot\,, \tag{1c}$$

The probabilities are then given by:

$$p(t|U,V,W) \propto p(v_1|t)\cdot p(v_2|t)\cdot p(v_3|t)\cdot p(t) \tag{2}$$

We use a flat prior $p(t)$ and model the velocity distributions as a gaussian which dispersion depends on the stellar age:

$$p(v_i|t) = \frac{1}{\sqrt{2\,\pi\,\sigma_i^2(t)}}\exp\left(-\frac{v_i^2(t)}{2\sigma_i^2(t)}\right) \tag{3}$$

The probabilities are then modelled by the velocities dispersions, the vertex deviation (ℓ_v), the asymmetric drift (considered in $V'_\odot(t)$) and the two other components of the Solar peculiar velocity. Figures 1 and 2 shows the relations fitted for these parameters as a function of age using data from the Geneva-Copenhagen survey (Casagrande et al. 2011).

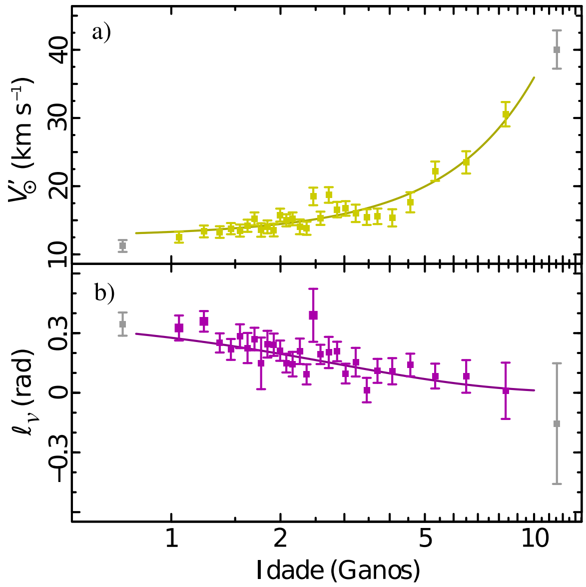

Figure 1. Relations fitted for $V'_\odot = 0.17t^2 + 0.63t + 12.5$ and $\ell_v(t) = 0.41\,\exp(-0.37t)$ using data from the Geneva-Copenhagen survey.

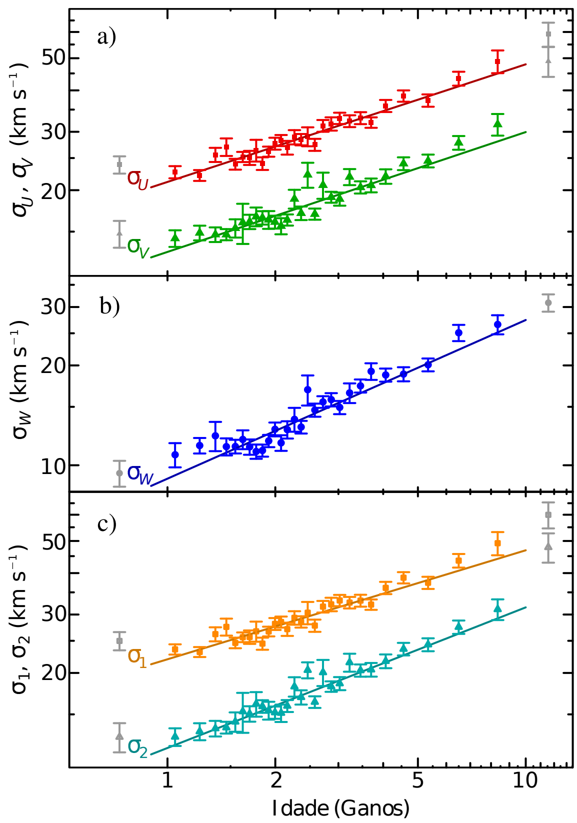

Figure 2. Power laws fitted for the velocity dispersions using data from the Geneva-Copenhagen survey: $\sigma_U = 21.2t^{0.35}$, $\sigma_V = 13.0t^{0.36}$, $\sigma_W = 9.1t^{0.48}$, $\sigma_1 = 22.0t^{0.33}$, $\sigma_2 = 11.9t^{0.42}$

3. Methodology

The Kinematical method described in the last section depends on how the stellar orbit changes with time, while the well established Isochronal Method is based on the internal evolution of the stars and the changes it causes on the observed atmospheric parameters. Therefore, both methods are physically independent. Despite the differences in physical causes and observables used, both methods follow a very similar statistical approach, and can be combined. The goal of this work is to test if this combination can be used to derive better defined ages.

Jørgensen & Lindegren (2005, hereafter J&L05) describe how the isochronal method can be used to derive a pdf for stellar ages through the calculation of what they call the G function and the prior age distribution $\phi(t)$:

$$f(t) \propto \phi(t)\,G(t) \tag{4}$$

The independence between both methods allows us to substitute the prior by the result of the Kinematical method, combining the information derived by the two methods.

$$\phi(t) \rightarrow p(t|U,V,W) \tag{5}$$

In order to calculate the G function, we developed a python module (called EITApy - **E**volutionary **T**racks and **I**sochrones: **T**ools and **A**pplications) to perform the numerical calculations described in Equations 4, 5 and 11 of J&L05. The evolutionary tracks data, needed for Eq. 5 in J&L05, was obtained using the PARSEC tracks (Bressan et al. 2012).

To quantify the increase of precision and accuracy obtained by this combination of the methods, we used EITApy to synthesize a sample of 10,000 stars of known ages from the PARSEC tracks. For each star, the sampled values of age, metalicity and mass are used to interpolate the evolutionary tracks and generate the stellar effective temperature, bolometric magnitude and surface gravity (Figura 3). We have also simulated the stellar velocities following the distributions fitted in the last Section (Figura 4). Due to the statistical nature of the kinematical method, we have simulated 10 sets of spatial velocities for each simulated star; we do this to lessen the influence of eventual outliers.

We have also artificially divided the stars in 7 groups according to their position in the HR diagram. The isochronal method results in better defined ages for the stars in the regions where the space between isochrones is larger, therefore, we expect the benefits of combining both methods to also depend on the stellar position in the diagram. We analyse the results for the different groups to find which one benefits the most by the combination with the kinematical method, and also check if in some cases the isochronal method alone returns better results.

We have applied the methods for all the simulated stars considering two sets of observational uncertainties: $(\sigma_{T_\mathrm{eff}}, \sigma_{M_\mathrm{bol}}, \sigma_{\log g}, \sigma_Z) = (40, 0.1, 0.1, 0.0005)$ and $(120, 0.3, 0.3, 0.0015)$. After obtaining the posterior age pdf, we characterize individual ages using both the most-likely age (t_ML) and the expected age (t_E):

$$t_\mathrm{ML} = \arg\max_t f(t)\,, \tag{6a}$$

$$t_\mathrm{E} = \int_{-\infty}^{\infty} t\,f(t)\,\mathrm{d}t\,, \tag{6b}$$

We are working with both definitions because they suffer from two different bias: t_ML is skewed towards the extremes of the age interval (0 and 13.7 Gyrs) because of the truncation of the pdf imposed by the prior distribution; and t_E is skewed towards the center of the distribution because of the spread of the pdf. Therefore, obtaining the same conclusions using both ages means the results are more robust and less affected by the bias in the definitions.

The comparison between the ages obtained by the combined methods and only by the isochronal method is made through the distribution of differences between estimated age and real age. If the mean of the distribution is far from zero, it means the method is biased; and if the spread is too large, it means that lots of stars are having their ages underestimated.

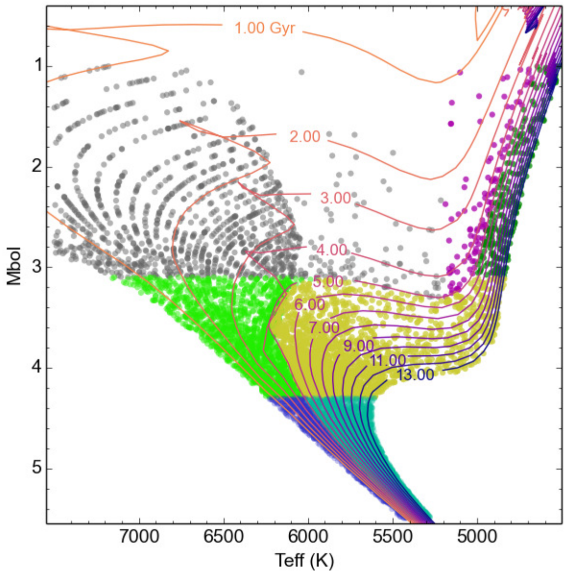

FIGURE 3. Distribution of the synthesized stars in the HR diagram. The different colors represent the 7 different groups to which the stars were assigned.

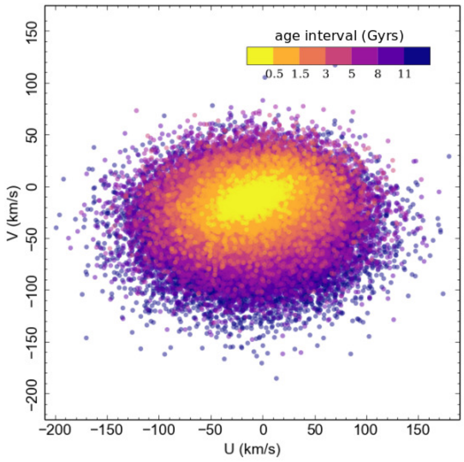

FIGURE 4. Distribution of the synthesized stars in the UV plane. The colors indicate different age intervals.

We also characterize the ages uncertainties using the spread of the pdf. In particular, we have defined the errors according to the ages of the 2.5% ($t_{2.5}$), 16% (t_{16}), 84% (t_{84}) and 97.5% ($t_{97.5}$) percentiles:

$$\delta_t = \frac{1}{4}\left[(t_{84} - t_{16}) + \frac{t_{97.5} - t_{2.5}}{2}\right], \quad (7)$$

this equation is designed to considerer both the spread closer to the median and at the tails, and its value is equal to 1σ for a normal distribution.

We have checked that our definition of δ_t can truly be used to compare the uncertainties between the combined method and the isochronal method by analysing the fraction of stars within $\pm1\delta_t$, $\pm2\delta_t$ and $\pm3\delta_t$ from the real age.

4. Results

We characterize the distributions of differences between estimated age and calculated age through its mean and standard deviation. When comparing two different methods, the one who

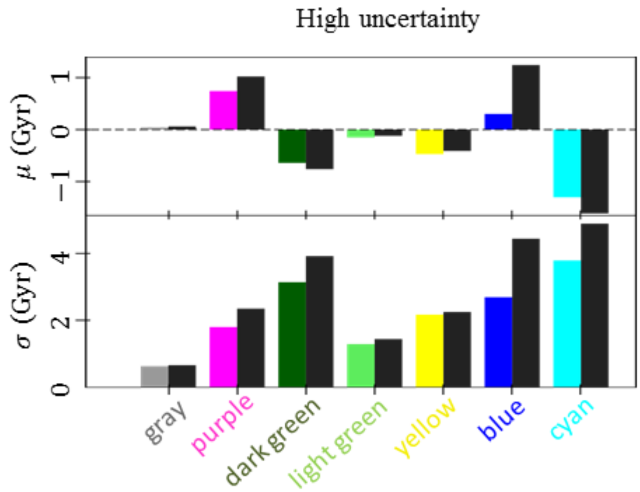

FIGURE 5. Characterizations of the distribution of differences between real age and estimated age (in the case of t_{ML} and high uncertainties), obtained by the combined method (groups colors) and isochronal method (dark gray)

performs better is the one with the mean closer to zero and the smaller dispersion. We characterized these distributions considering our two defined ages (t_{ML} and t_{E}) and the two sets of observational uncertainties. The results are presented in Table 1.

It's clear that the two age definitions are affected by different age's bias. Both the combined and the isochronal method are, on average, more precise when considering the most likely age instead of the expected age: the mean of the distributions are closer to zero. Although, when considering accuracy, the expected age is the one that produce the best results, as for both methods the dispersion are systematically lower. This results hold true for both observational uncertainties scenarios and most of the groups.

Figure 5 displays the means and dispersions for the age's differences obtained by the combined method (groups colors) and isochronal method (dark gray), using t_{ML}, for the high uncertainty scenario. As we can see, the combined method systematically returns means that are closer to zero and smaller dispersions. It indicates that the combined method returns ages that are both more accurate and more precise. Although, it's important to notice that the p-value only allows us to reject the null hypotenuses (that both distributions are equivalent), with a 5% significance level, for the case of the light green, blue and cyan groups.

In general though, the differences obtained by the combined method are only significant in the cases of the blue and cyan groups: which corresponds to low mass stars that are at the main sequence or close to the turn off. In the other cases, the combined method either improves the ages by a small amount, or display no changes at all, which is also important, because in none of the considered scenarios the combined method returns results that are worse than the isochronal method alone. It means that the combined method can be applied to a sample containing both low mass main sequence stars and evolved stars, without having to worry if it is negatively effecting the ages of some stars.

The results obtained using t_{E} and it's average with t_{ML}, for the scenarios of low and high uncertainties are only displayed in Table 1. In all cases, the results are similar to what was discussed above. Considering that t_{E} and t_{ML} are affected by two different bias, obtaining the same results using both of them indicates that the our conclusion is robust, and the combined method in fact

Table 1. Characterization of the distribution of differences between the known simulated age and the estimated age by the combined method (index C) and the isochronal method (index I). Results are shown for both uncertainties scenarios (low and high) and for three age estimators: the most-likely age (t_{ML}), the expected age (t_{E}) and the average between them (t).

Group	Uncertainty	t_{MP}					t_{E}					t				
		μ_{C}	μ_{I}	σ_{C}	σ_{I}	p	μ_{C}	μ_{I}	σ_{C}	σ_{I}	p	μ_{C}	μ_{I}	σ_{C}	σ_{I}	p
Grey	Low	-0.01	-0.01	0.24	0.25	1.0e+00	0.03	0.03	0.24	0.24	8.6e-01	0.01	0.01	0.23	0.24	8.7e-01
	High	0.04	0.06	0.63	0.66	1.0e+00	0.16	0.15	0.66	0.71	2.5e-02	0.10	0.10	0.63	0.66	4.6e-01
Purple	Low	0.13	0.14	0.75	0.76	1.0e+00	0.48	0.51	0.83	0.89	9.7e-01	0.31	0.32	0.78	0.81	1.0e+00
	High	0.74	1.02	1.80	2.35	9.4e-01	2.19	2.67	1.49	1.55	2.1e-04	1.47	1.84	1.58	1.84	1.0e-01
Dark Green	Low	-0.13	-0.15	1.95	2.11	9.5e-01	-0.15	-0.16	1.66	1.74	9.7e-01	-0.14	-0.15	1.75	1.86	7.9e-01
	High	-0.64	-0.76	3.14	3.92	1.4e-01	-0.47	-0.65	2.25	2.44	1.0e-01	-0.56	-0.70	2.54	2.93	3.1e-01
Light Green	Low	0.01	-0.01	0.68	0.72	1.3e-01	0.05	0.01	0.58	0.62	1.0e-02	0.03	-0.00	0.61	0.65	4.4e-02
	High	-0.15	-0.12	1.29	1.44	2.3e-02	0.20	0.12	1.06	1.13	1.0e-04	0.03	0.00	1.14	1.23	1.2e-03
Yellow	Low	-0.03	-0.03	0.80	0.82	1.0e+00	-0.08	-0.08	0.76	0.78	1.0e+00	-0.06	-0.06	0.77	0.79	9.7e-01
	High	-0.47	-0.41	2.17	2.25	3.4e-01	-0.61	-0.79	1.73	1.82	1.2e-07	-0.54	-0.60	1.89	1.96	5.7e-03
Blue	Low	0.10	0.24	1.96	2.44	2.5e-07	0.86	0.96	1.57	1.75	3.8e-02	0.48	0.60	1.72	2.04	9.9e-04
	High	0.30	1.24	2.69	4.44	2.6e-38	1.88	2.57	1.81	1.95	4.3e-25	1.09	1.90	2.17	3.02	8.0e-26
Cyan	Low	-0.34	-0.34	2.47	2.68	2.8e-02	-0.70	-0.98	1.89	2.03	7.6e-13	-0.52	-0.66	2.13	2.29	3.8e-04
	High	-1.30	-1.61	3.79	4.89	3.4e-29	-1.38	-2.20	2.45	2.42	4.2e-65	-1.34	-1.91	3.02	3.37	6.9e-27

returns better results for the blue and cyan groups, no matter how you choose to characterize the age from the pdf.

5. Conclusions

We have developed a new method to derive an age pdf from the stellar spatial velocities. To do that, we calibrate the velocities distribution as a function of age using data from the GCS survey.

We show how this kinematical age pdf can be incorporated to the isochronal method, using more information to better constrain the stellar age.

In order to validate this combination, we have synthesized a sample of 10 000, with known ages and, applied to them the combined method and the isochronal method alone. The comparison of the results allows us to conclude that:

- The combined method considerably improves the precision and accuracy for the low mass stars that are at the main sequence or close to the turn off.
- The increase in accuracy and precision are more evident when observational uncertainties for the atmospheric parameters are higher.
- In none of the considered scenarios the combined method results in worse defined ages than the isochronal method alone. It's either better defined or statistically equivalent.

The low mass stars, that benefits the most by the combination of the isochronal and kinematical methods are also the best tracers of the Galactic evolution, due to their extended lifetime. Also, the high uncertainty scenario, in which the differences between the combined and isochronal method are greater, is the one that represents the quality of data in large surveys. It's these large surveys that contains enough stars for us to derive information about the chemodynamical evolution of the Galaxy. Therefore, combined with precise astrometric information from Gaia (for the determination of the spatial velocities), the method derived here has great potential to improve the age determinations for stars from surveys like RAVE (Steinmetz et al. 2006) and consequently allows for a better understanding of the chemodynamical evolution of our Galaxy.

References

Casagrande L., et al., 2011, A&A, 530, A138
Nordström B., et al., 2004, A&A, 418, 989
Jørgensen B. R. & Lindegren L., 2005, A&A, 436, 127
Bressan A., et al., 2012, MNRAS, 427, 127
Steinmetz M., et al., 2006, AJ, 132, 1645