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Bayesian Analysis of CCDM models

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Abstract. This work tests six different spatially flat models for Creation of Cold Dark Matter (CCDM) using three statistical criteria, in light of SNe Ia data: Akaike Information Criterion (AIC) (Akaike 1974), Bayesian Information Criterion (BIC) and Bayesian Evidence (BE) (Hobson et al. 2010). These criteria allow to compare models considering goodness of fit and number of free parameters, penalizing excess of complexity. We find that JO model is slightly favoured over LJO/ΛCDM model, however, neither of these, nor $\Gamma = 3\alpha H_0$ model can be discarded from the current analysis. Three other scenarios are discarded either because poor fitting or because of the excess of free parameters.

Resumo. Este trabalho testa seis modelos planos para Criação de Matéria Escura Fria (CCDMs em inglês) usando três critérios estatísticos diferentes, à luz de dados de SNe Ia: Akaike Critério de Informação (AIC) (Akaike 1974), Critério Bayesian de Informação (BIC) e Evidência Bayesian (BE) (Hobson et al. 2010). Estes critérios permitem comparar modelos considerando a qualidade do ajuste e o número de parâmetros e o excesso de complexidade. Encontramos que o modelo JO é levemente favorável em relação ao modelo LJO/ΛCDM, entretanto nenhum deles pode ser descartado nem o modelo $\Gamma = 3H_0$ nesta análise. Três outros cenários são descartados por causa da qualidade do ajuste e do excesso de parâmetros. O método de aumento da Evidência Bayesian na re-parametrização em ordem de reduzir a degenerescência dos parâmetros também é desenvolvida.

Keywords. Cosmology: observations – dark matter – cosmological parameters

1. Creation of Cold Dark Matter (CCDM) Models

Considering the process of matter creation, we have a source term at the level of the Einstein's Field Equations:

$$\frac{\dot{\rho}_{dm}}{\rho_{dm}} + 3\frac{\dot{a}}{a} = \Gamma, \tag{1}$$

where Γ is creation of dark matter ratio in units of s^{-1}. The creation of CDM is related with negative pressure p_c, by assuming an "adiabatic" creation, i.e., the entropy per particle as constant. Adiabatic regime is a hypothesis in which only the source of entropy grows up with the creation of matter from Universe. Explicitly the expression is:

$$\dot{\sigma} = \frac{\Psi}{nT}\left(\beta - \frac{\rho + p}{n}\right) \tag{2}$$

where σ is entropy per particle, Ψ is the numerical rate of creation of particles, n is the density of particles, T is temperature and β emerges like an assumption of creation pressure:

$$p_c = -\frac{\beta\Psi}{\Theta} \tag{3}$$

where $\Theta = 3H$ is the expansion rate of the mass and $H \equiv \dot{a}/a$. So, in case $\dot{\sigma} = 0$, as was assumed, we found $\beta = \frac{\rho+p}{n}$, so the creation pressure is given by:

$$p_c = -\frac{\rho + p}{\Theta}\frac{\Psi}{n} = -(\rho + p)\frac{\Gamma}{3H}. \tag{4}$$

We can see that the dynamics of the Universe is directly affected by the rate of creation of cold dark matter.

Table 1. Model parameters and priors. Concerning background equations, M_1 (LJO) is indistinguishable with ΛCDM.

Model	Creation rate	Reference
M_0	$\Gamma = \frac{3\alpha H_0^2}{H}$	Jesus & Andrade-Oliveira (2016) (JO)
M_1	$\Gamma = 3\alpha\frac{\rho_{c0}}{\rho_{dm}}H$	Lima et al. (2010) (LJO)
M_2	$\Gamma = 3\alpha H_0$	Graef et al. (2014)
M_3	$\Gamma = 3\beta H$	–
M_4	$\Gamma = 3\alpha H_0\left(\frac{H_0}{H}\right)^n$	–
M_5	$\Gamma = 3\alpha\frac{H_0^2}{H} + 3\beta H$	Graef et al. (2014)

2. Models

The models studies here are briefly described on Table 1.

In all models analysed here we have taken into account the contribution of baryons. The baryon density was assumed to be a fixed parameter, given by Planck satellite as $\Omega_b = 0.049$. The flat universe was chosen as indicated by CMB and preferred by inflation hypothesis, then: $\Omega_k \equiv 0$.

3. Methodology

Our methodology involved Bayesian model selection. The likelihood function is a main ingredient in this kind of analysis, other important aspect is the Ockham's razor principle. Next we summarise the model selection criteria used here.

Table 2. Results of the model selection analysis for the different models.

Model	χ^2_{min}	χ^2_ν	ν	ΔAIC	ΔBIC	V_P	$\ln B_{i0}$
$M_0 : \Gamma = \frac{3\alpha H_0^2}{H}$	562.251	0.97107	1	0	0	1	0
$M_1 : \Gamma = 3\alpha \frac{\rho_{c0}}{\rho_{dm}} H$	562.227	0.97103	1	-0.024	-0.024	1	0.043
$M_2 : \Gamma = 3\alpha H_0$	563.131	0.97259	1	0.880	0.880	1	0.155
$M_3 : \Gamma = 3\beta H$	566.936	0.97916	1	4.685	4.685	1	0.955
$M_4 : \Gamma = 3\alpha H_0 \left(\frac{H_0}{H}\right)^n$	562.220	0.97270	2	1.969	6.332	20	0.921
$M_5 : \Gamma = 3\alpha \frac{H_0^2}{H} + 3\beta H$	562.213	0.97269	2	1.962	6.325	20	1.463

3.1. Akaike Information Criterion

Akaike Information Criterion (AIC) provides a relative measure of the quality of models to describe a given data set. This tool emerges from the Information Theory (Akaike, 1974).

$$\mathrm{AIC} = -2 \ln \mathcal{L}_{max} + 2p. \tag{5}$$

where $\mathcal{L}_{max}$ is the maximum likelihood and p is the number of free parameters of the model. In our case, we have: $\mathcal{L} = \mathcal{N} \exp\left(-\frac{\chi^2}{2}\right)$, where $\mathcal{N}$ is a normalisation constant. So, in our case, the AIC is given by

$$\mathrm{AIC} = \chi^2_{min} - 2 \ln \mathcal{N} + 2p. \tag{6}$$

3.2. Bayesian Information Criterion

Bayesian Information Criterion (BIC) (Schwarz, 1978) is an interesting manner to compare different models with different number of parameters. BIC penalises models in agreement with Occham's razor principle: simple models, which equally describe the phenomenon, are preferred over complex models.

$$\mathrm{BIC} = -2 \ln \mathcal{L}_{max} + p \ln N. \tag{7}$$

where N is number of data, $\mathcal{L}_{max}$ is maximum likelihood function and p is number of free parameters. In our case, BIC is given by:

$$\mathrm{BIC} = \chi^2_{min} - 2 \ln \mathcal{N} + p \ln N. \tag{8}$$

3.3. Bayesian Evidence

Bayesian Evidence (BE) emerges from Bayes' theorem and it is the integral of the posterior in all parameter space.

$$B_{ij} = \frac{E(M_j)}{E(M_i)}. \tag{9}$$

The Bayesian Evidence is the most complete criterion used here, as it contains information of the full posterior and is in agreement with the Occham's razor. BIC is an useful approximation of BE, as BE usually is hard to obtain.

3.4. Models selections and matter creation

AIC, BIC and BE are calculated for all models studied here. The results are in Table 2.

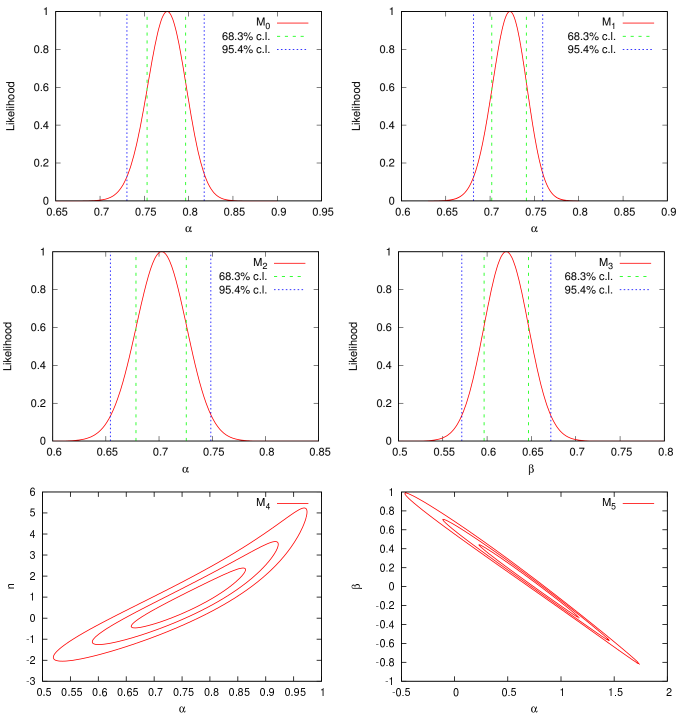

Figure 1. The results of our statistical analysis, with constraints from SNe Union 2.1 data (**?**). **Panels (a)-(d)** Likelihoods for the parameters on each indicated model, M_0-M_3, including 68.3% and 95.4% confidence levels. **Panels (e)-(f)** Contours for 68.3%, 95.4% and 99.7% confidence intervals for each indicated model, M_4 and M_5.

4. Conclusion

We have compared 6 spatially flat CCDM models, including one that is degenerate with the ΛCDM model. The JO model is slightly favoured over ΛCDM in the Bayesian evidence, however, ΛCDM and $\Gamma = 3\alpha H_0$ can not be discarded from this analysis. Models M_3 and M_4 are moderately weak and M_5 can certainly be discarded with the current parameterization. At this point, it is important to mention that JO model is equivalent to the late phase. Further investigations of CCDM models may include spatial curvature, other background data and the evolution of density perturbations.

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